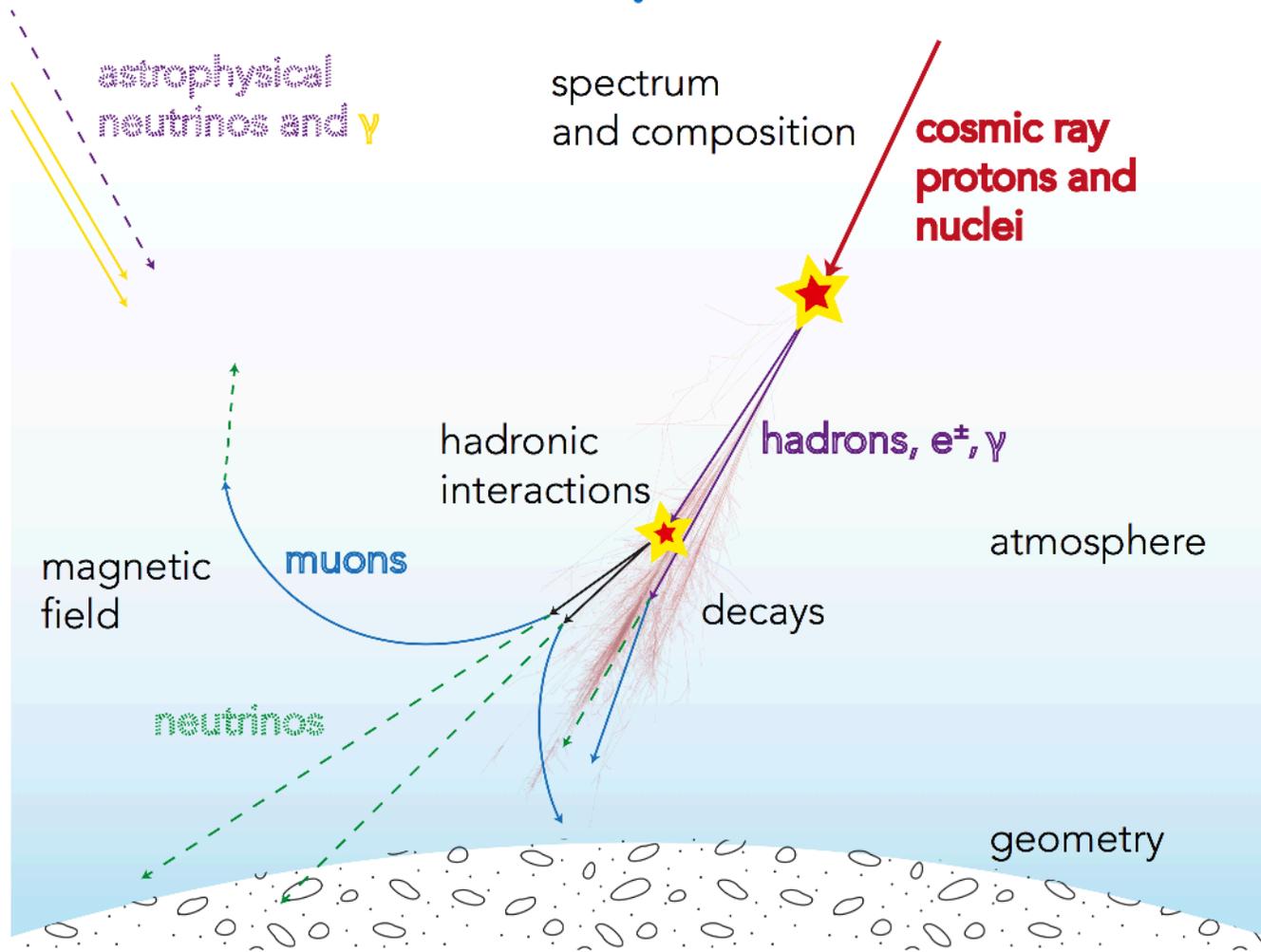


Atmospheric leptons



> For high precision calculations all phenomena need accurate modeling

> Not so well known “ingredients”:

- Cosmic ray spectrum and composition
- Hadronic interactions
- Atmosphere (dynamic, depends on use case)
- (Rare) decays (solved)

> Energy range MeV – EeV!

Production channels

conventional

$$p, A + \text{air} \rightarrow \pi^\pm, \pi^0, K^\pm, K_{S,L}^0$$

muons and muon neutrinos

$$\pi^\pm, K^\pm \rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu)$$

electron neutrinos

$$K^\pm, K_L^0 \rightarrow [\pi^\pm, \pi^0] e^\pm \nu_e (\bar{\nu}_e)$$

prompt

$$p, A + \text{air} \rightarrow D, \Lambda_C \rightarrow \nu_\mu, \nu_e, \mu$$

Subset of dominant decay channels

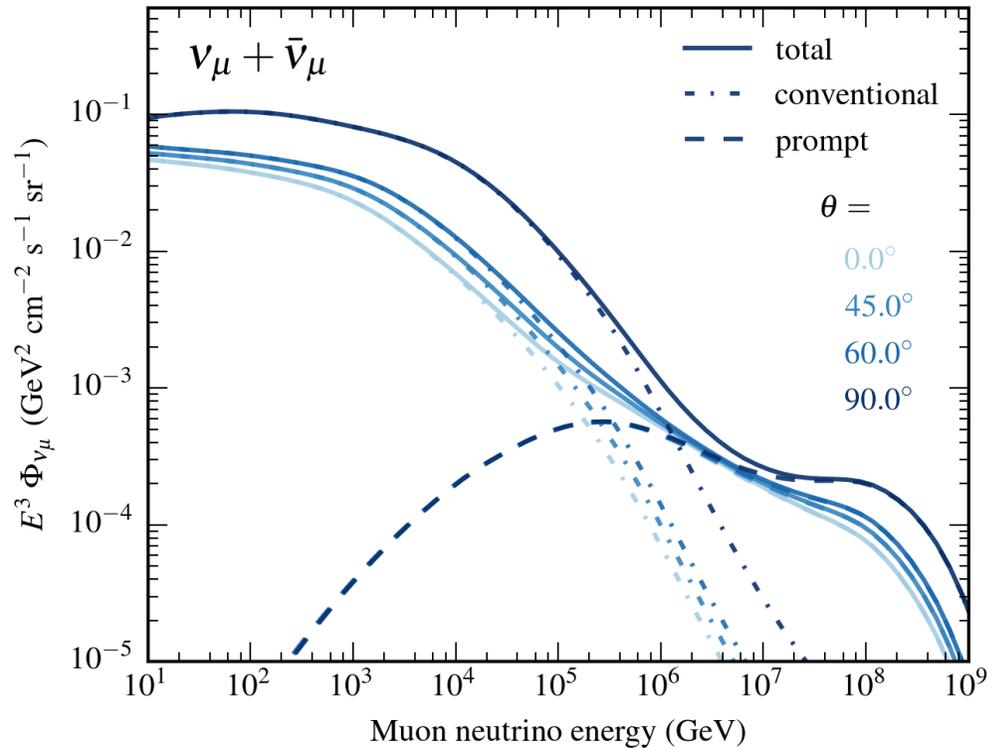
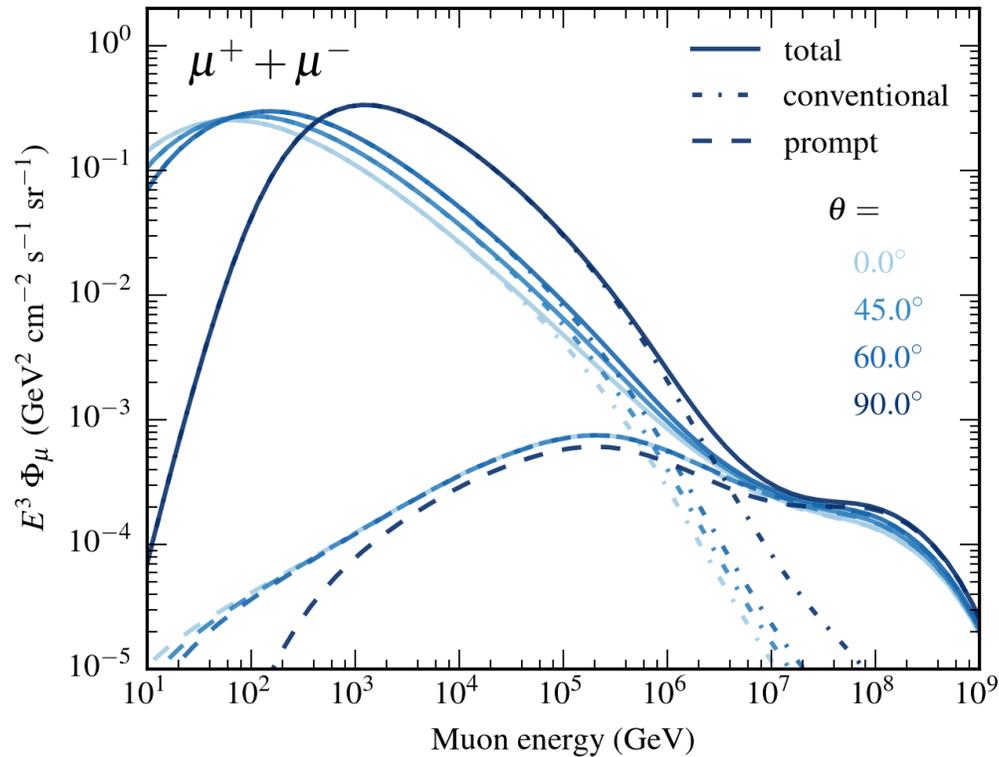
decay channel	branching ratio (BR)
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$	100 %
$\pi^+ \rightarrow \mu^+ \nu_\mu$	99.9877 %
$K_{e3}^0 : K_L^0 \rightarrow \pi^\pm e^\mp \nu_e$	40.55 %
$K_{\mu3}^0 : K_L^0 \rightarrow \pi^\pm \mu^\mp \nu_\mu$	27.04 %
$K^+ \rightarrow \mu^+ \nu_\mu$	63.55 %
$K_{e3}^+ : K^+ \rightarrow \pi^0 e^+ \nu_e$	5.07 %
$K_{\mu3}^+ : K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$	3.353 %
$D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$	9.2 %
$D^0 \rightarrow K^- \mu^+ \nu_\mu$	3.3 %

+ charge conjugates

<http://pdg.lbl.gov>



General shape of high energy lepton spectra



> 30 km

Depth X
(g/cm^2)

Zenith angle changes competition between decay and (re-)interaction



Transport equations (hadronic cascade equations)

System of non-linear PDE for each particle species h ($\sim 62 \times \#E\text{-bins}$) :

$$\frac{d\Phi_h(E, X)}{dX} = - \frac{\Phi_h(E, X)}{\lambda_{\text{int},h}(E)} \quad \text{cosmic ray physics} \quad \text{interactions with air}$$

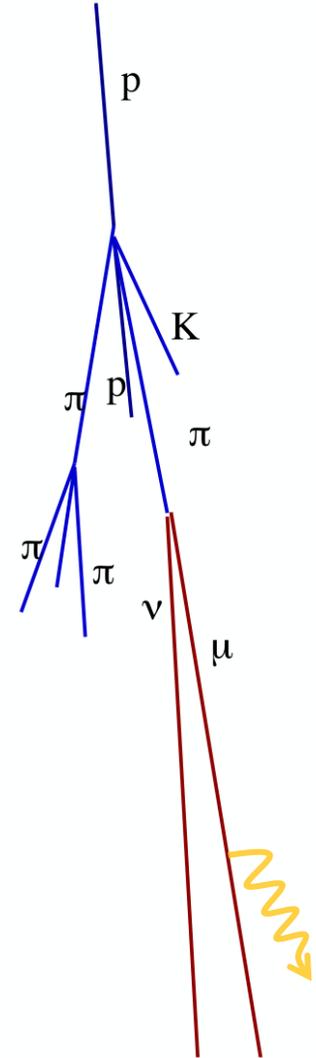
$$- \frac{\Phi_h(E, X)}{\lambda_{\text{dec},h}(E, X)} \quad \text{atmospheric physics} \quad \text{Decays}$$

$$- \frac{\partial}{\partial E} (\mu(E) \Phi_h(E, X)) \quad \text{Energy losses (radiative)}$$

$$+ \sum_k \int_E^\infty dE_k \frac{dN_{k(E_k) \rightarrow h(E)}}{dE} \frac{\Phi_k(E_k, X)}{\lambda_{\text{int},k}(E_k)} \quad \text{Re-injection from interactions}$$

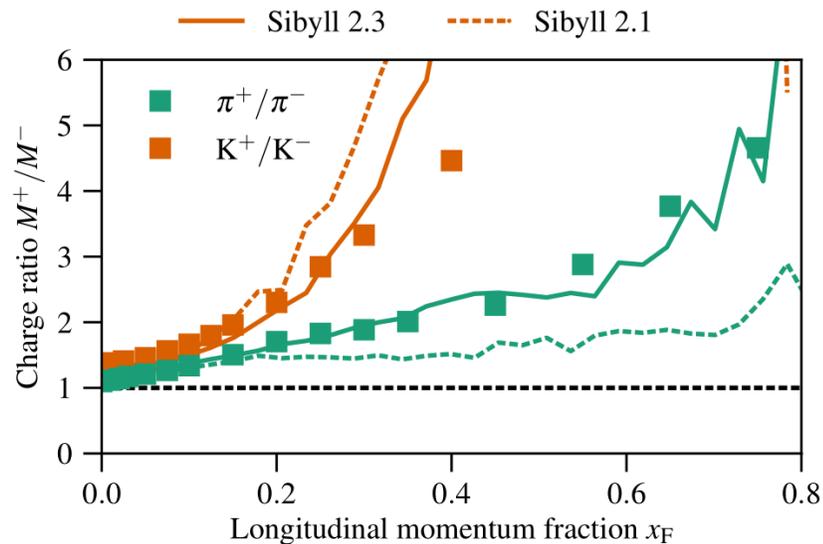
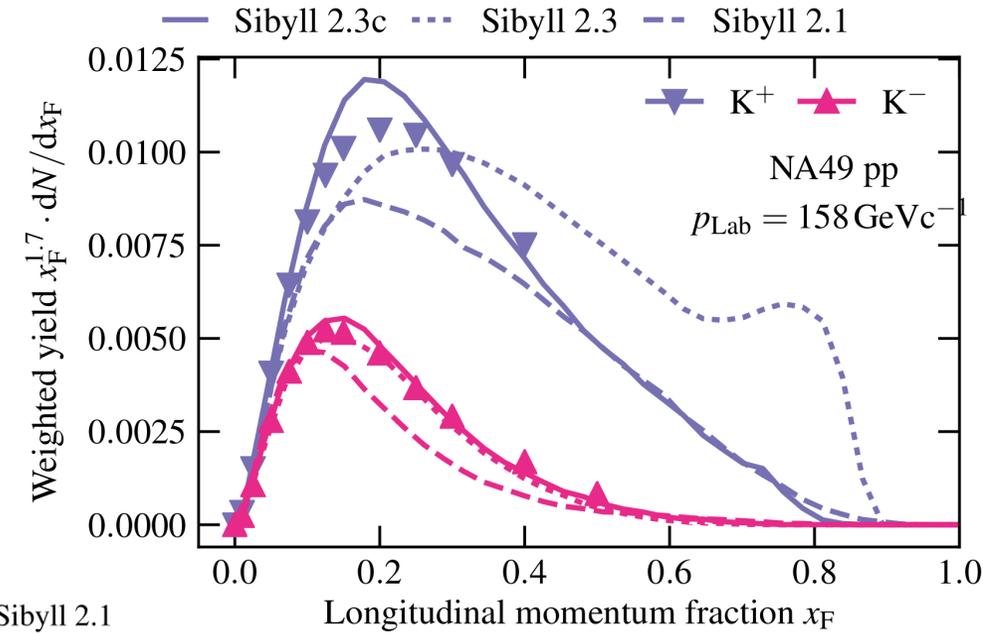
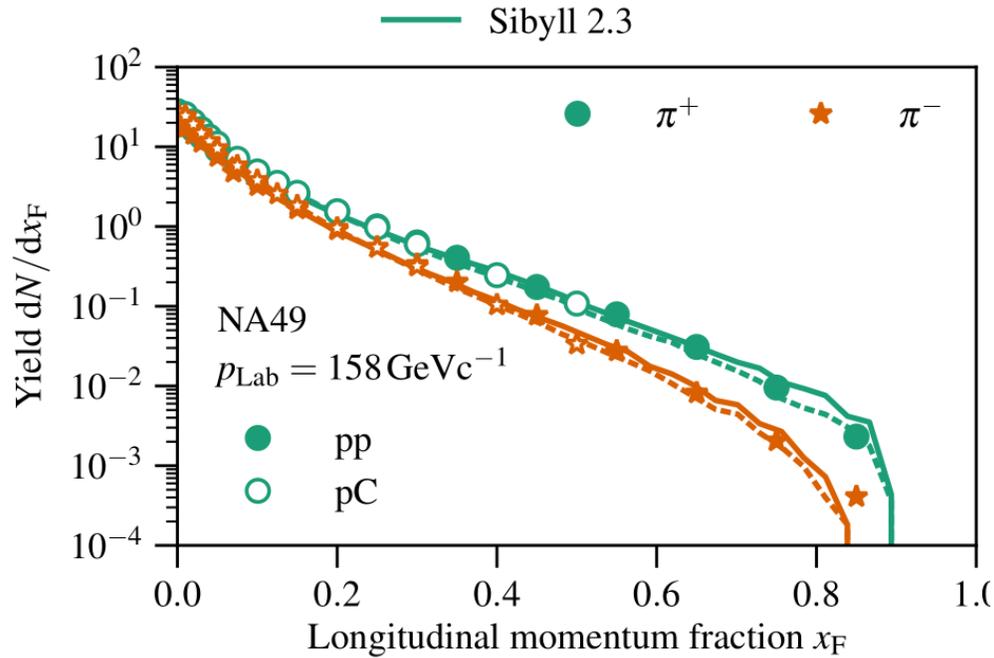
$$+ \sum_k \int_E^\infty dE_k \frac{dN_{k(E_k) \rightarrow h(E)}^{\text{dec}}}{dE} \frac{\Phi_k(E_k, X)}{\lambda_{\text{dec},k}(E_k, X)} \quad \text{Re-injection from decays}$$

$$\text{particle physics}$$



Inclusive secondary particle distributions (as in fixed-target experiment)

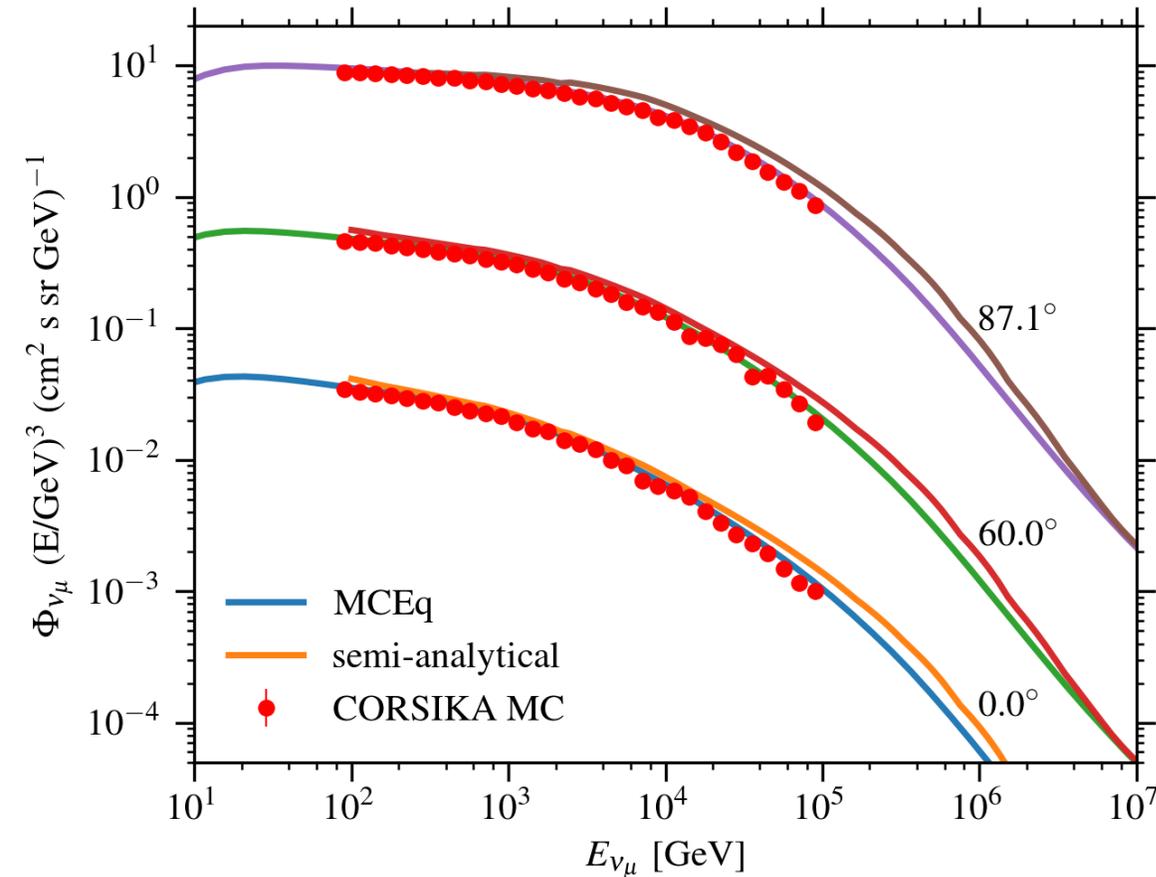
Measurements at low energy,
no clear extrapolation recipe



No kaon data for
proton-carbon!



MCEq: open-source Python code



- Simultaneous solution of up to 8000 kinetic equations
- Energy range 1 (30) GeV – 10^{11} GeV
- All models included
- High optimization: multi-core, GPU, ... (BLAS, MKL, CUDA) (~milli-seconds!)
- MIT licensed @ <https://github.com/afedynitch/MCEq>

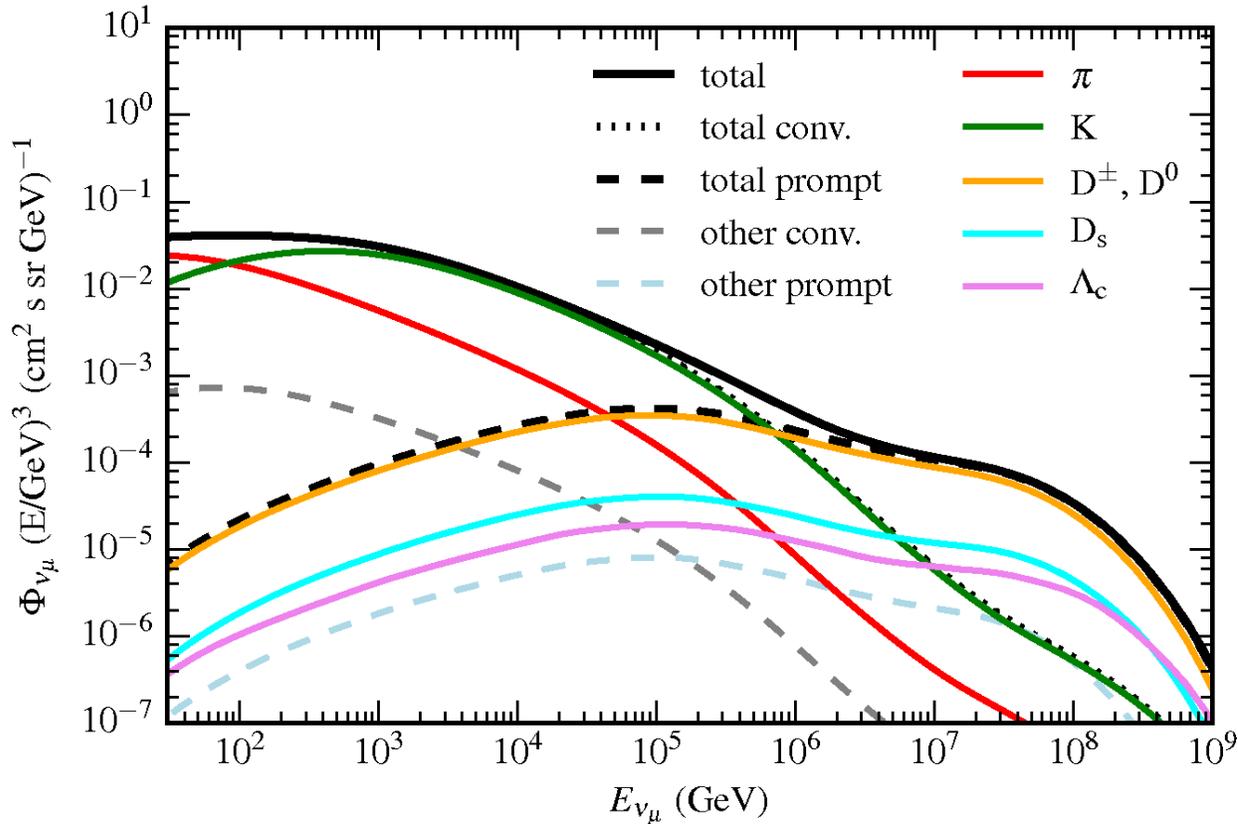
CORSIKA: A. Fedynitch, J. Becker Tjus and P. Desiati, PRD 2012

MCEq: A. Fedynitch, R. Engel, T. K. Gaisser, F. Riehn and S. Todor. PoS ICRC 2015, 1129



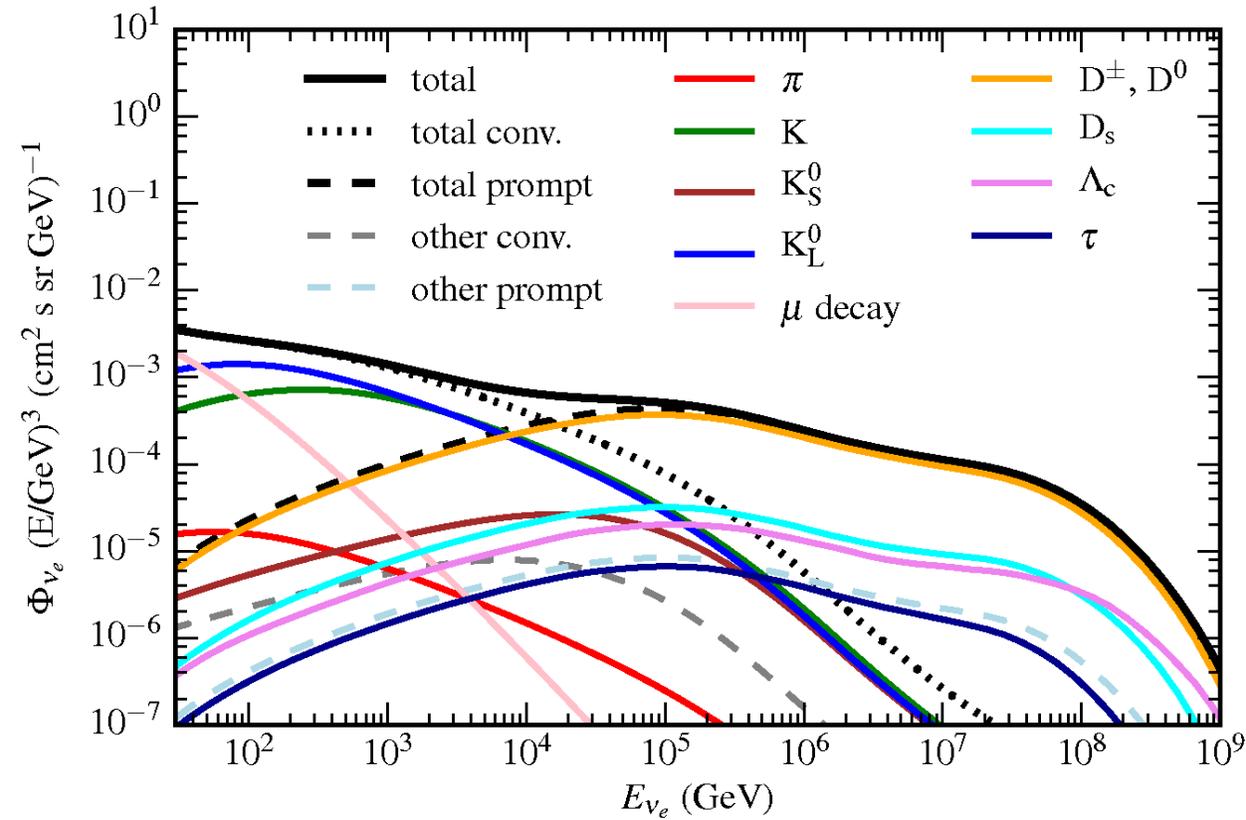
Relevant hadrons for neutrino production

Muon neutrinos



pion decay kaon decay charm decay

Electron neutrinos

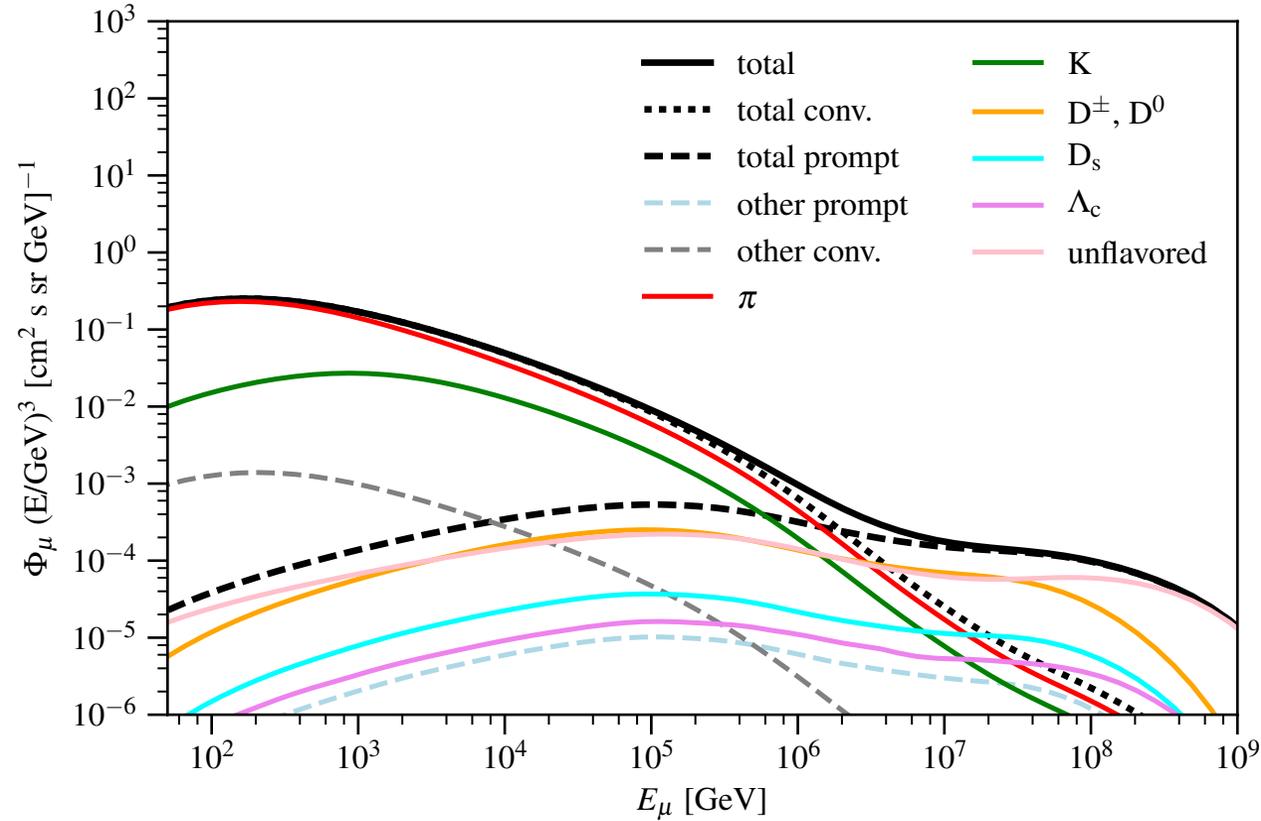


muon decay kaon decay charm decay



Relevant particles for muon production

Muons

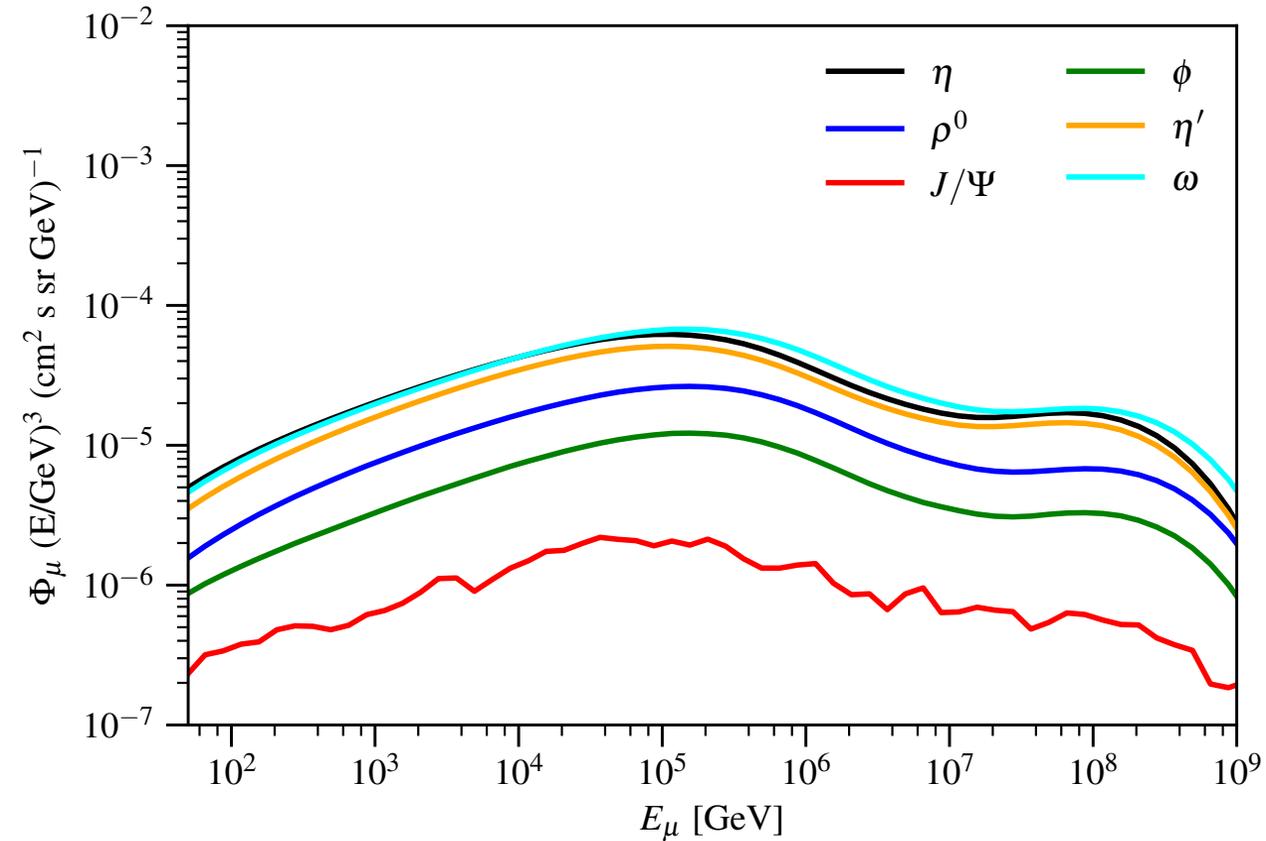


pions + kaons

charm + unflv.

unflavored

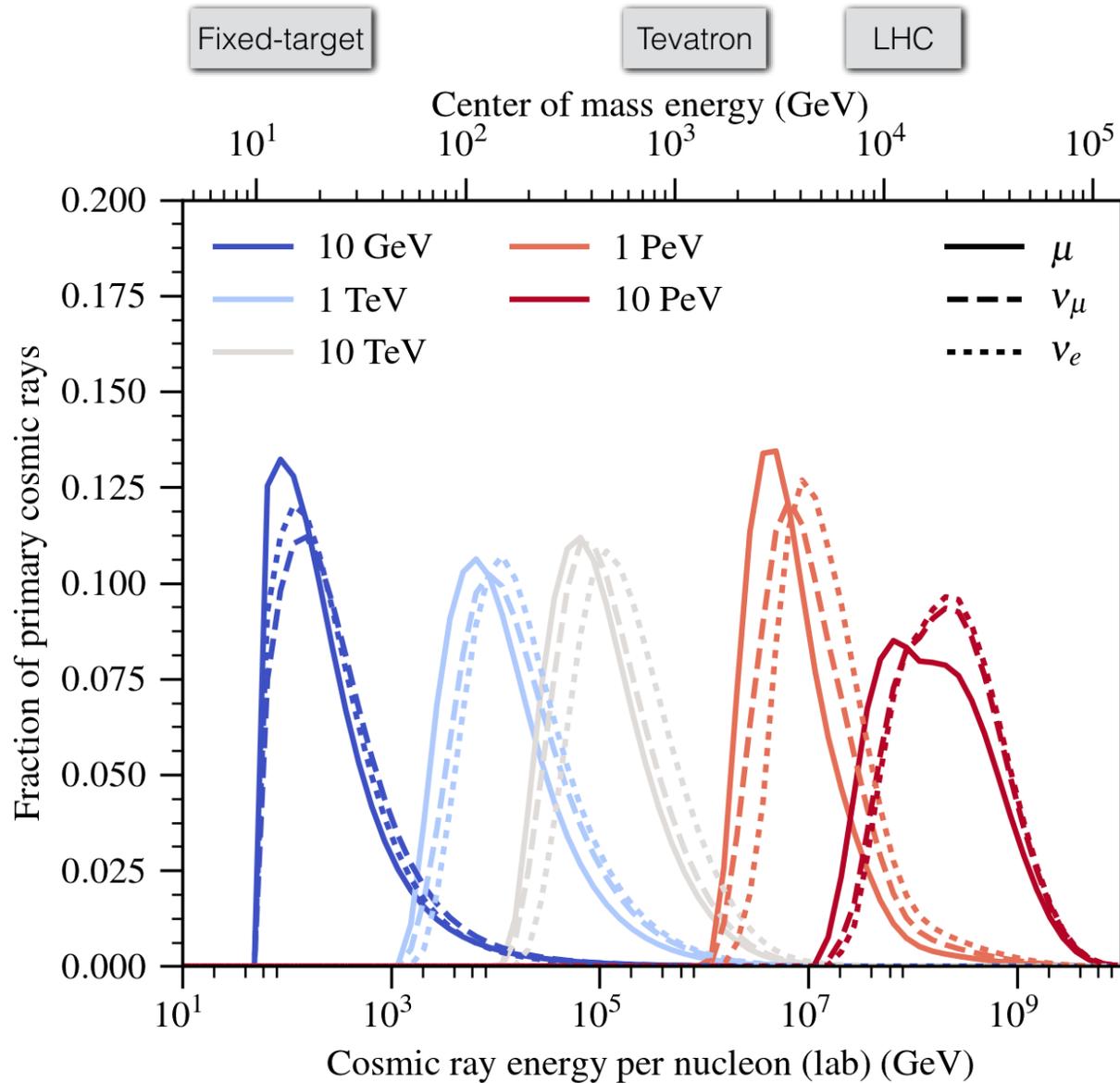
..from unflavored mesons



These mesons decay into muon pairs with small branching ratios



Relation between lepton and cosmic ray energy



Energy range covered by particle accelerators.



If energy is not a problem...

Kinematic variables

$$\theta = \arctan \frac{p_T}{p_z}$$

$$\eta = -\ln \left(\tan \frac{\theta}{2} \right)$$

$$x_{\text{lab}} = \frac{E_{\text{secondary}}}{E_{\text{primary}}} \approx \frac{p_{z,\text{secondary}}}{E_{\text{primary}}}$$

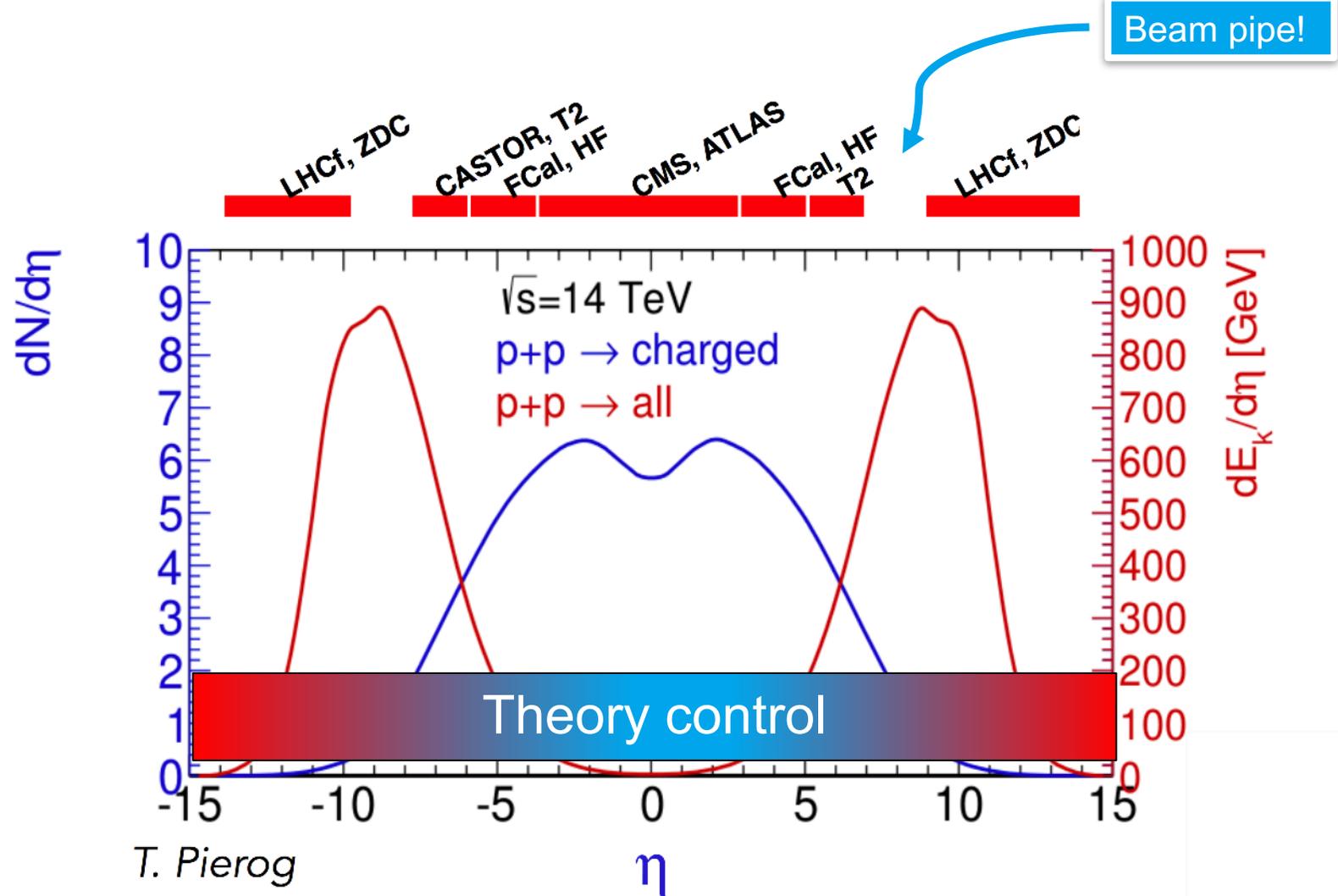
For atmospheric leptons

$$p_z \sim \text{TeV} - \text{PeV}$$

$$p_T \sim \text{few GeV}$$

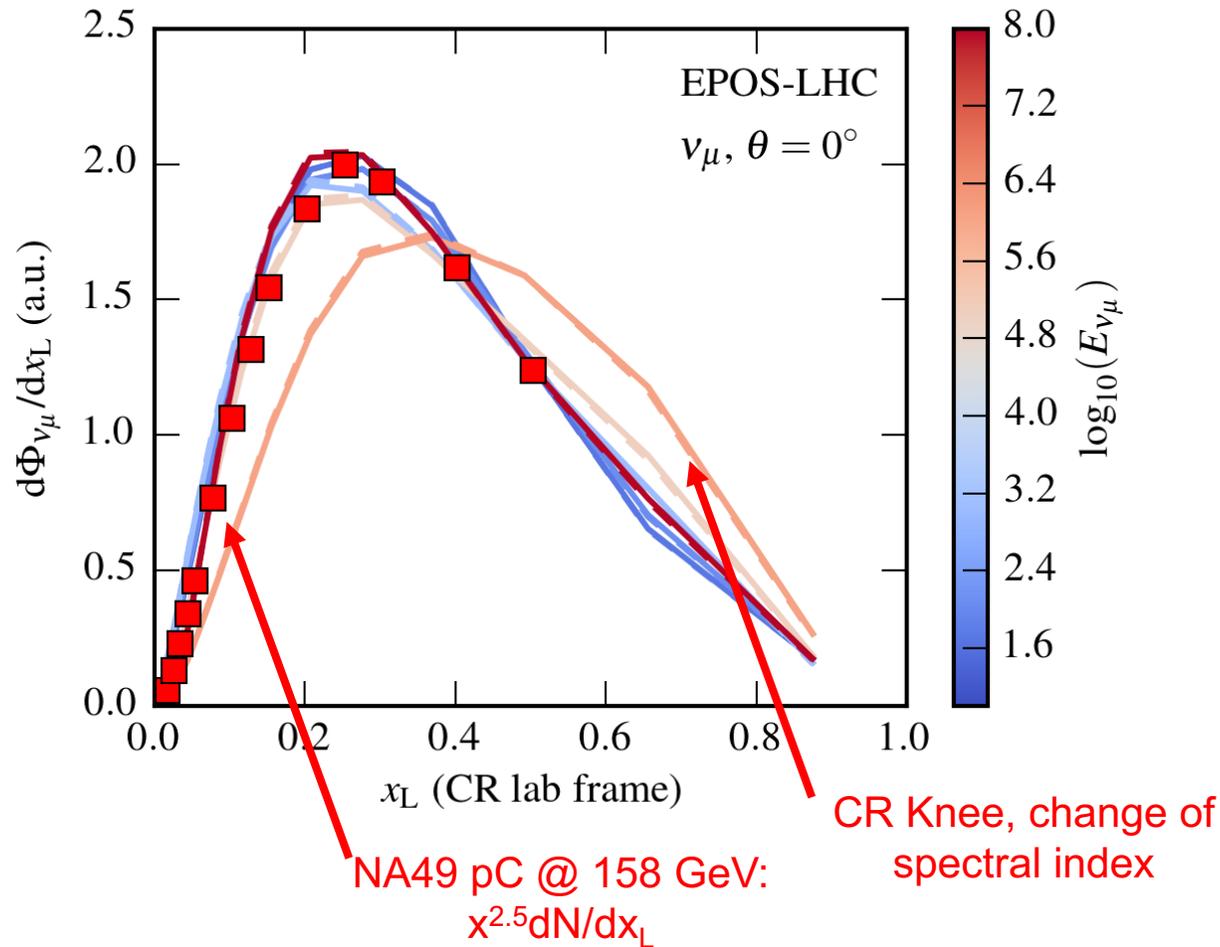
$$\theta \sim \mu\text{rad}$$

$$x_{\text{lab}} > 0.1, \quad \eta \rightarrow \infty$$



Secondary hadron (pion/kaon/charm) energy fractions

x_{lab} regions contributing to inclusive muon neutrino flux

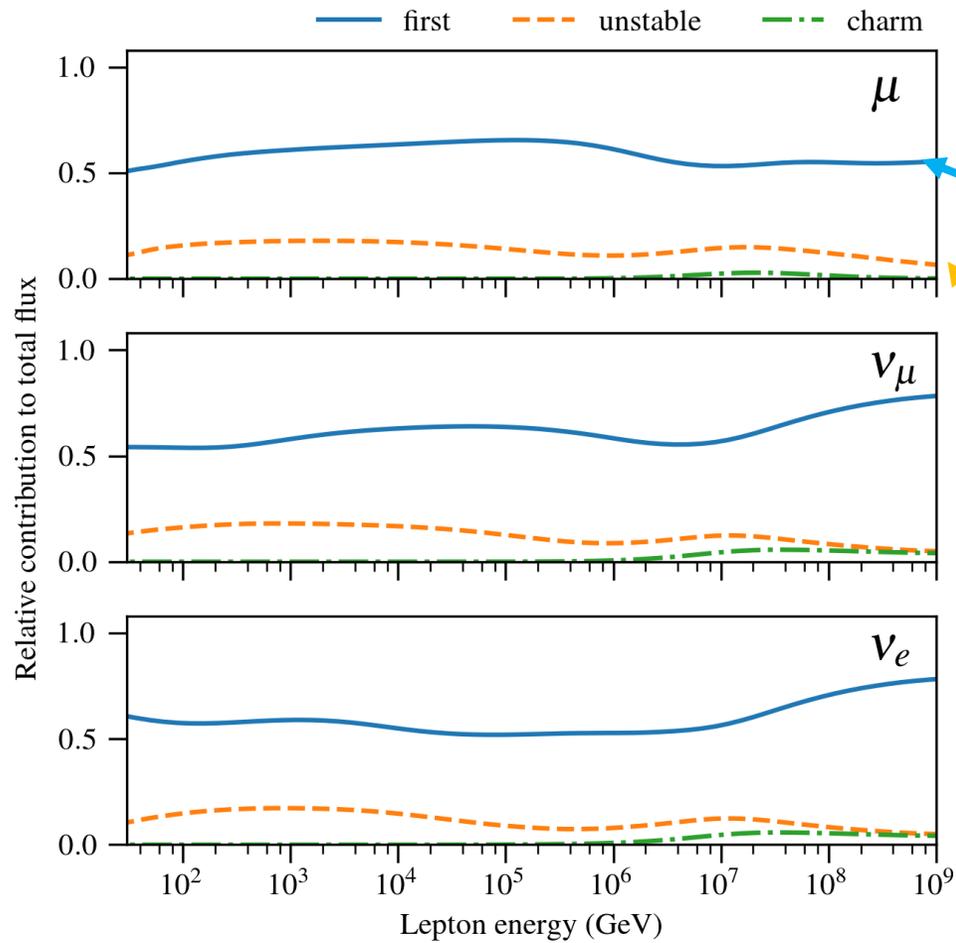


- > Atmospheric leptons are sensitive mostly to $x_{\text{lab}} \sim 0.2$ and above
- > Reason: steepness of primary CR spectrum

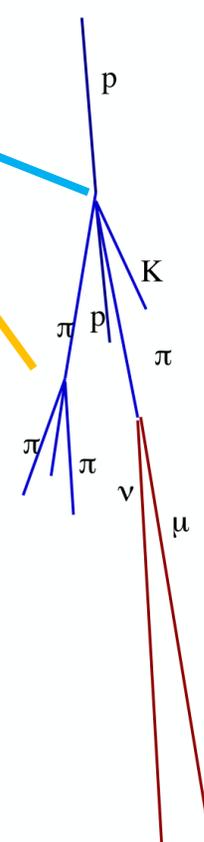
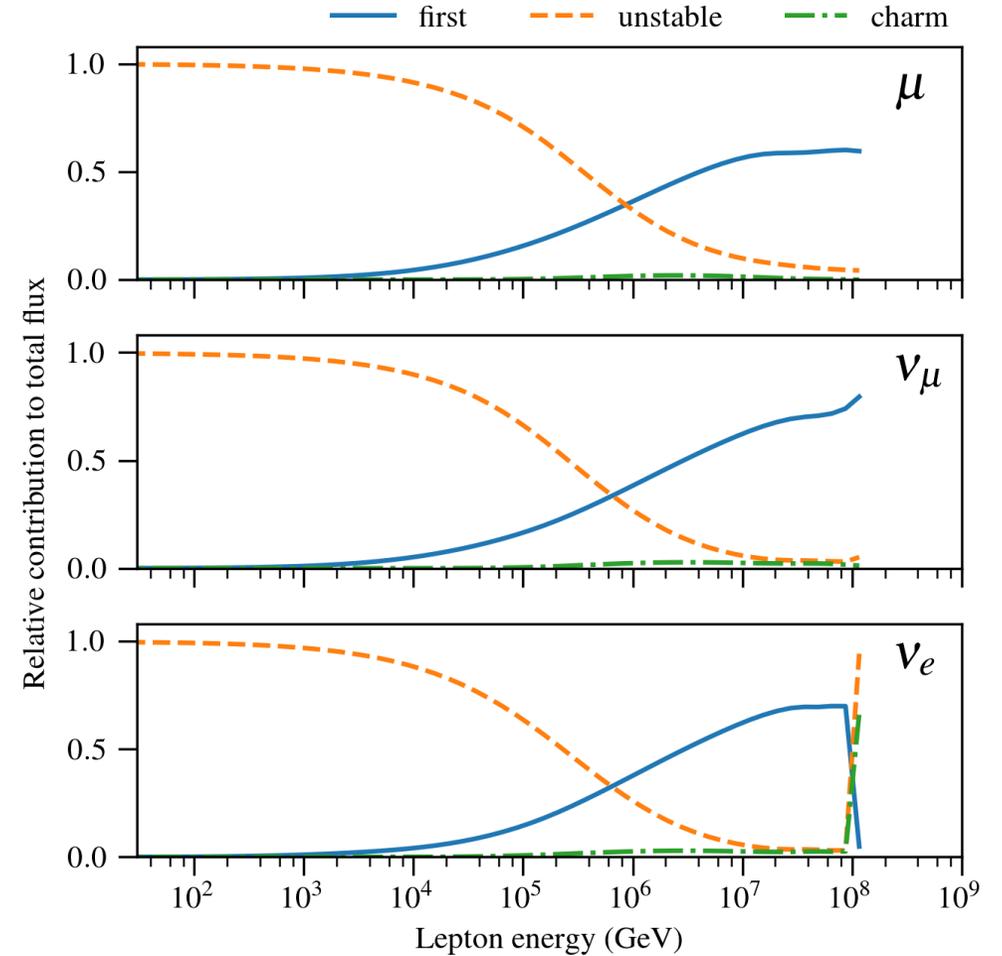


Role of secondary interactions in cascade

Inclusive leptons



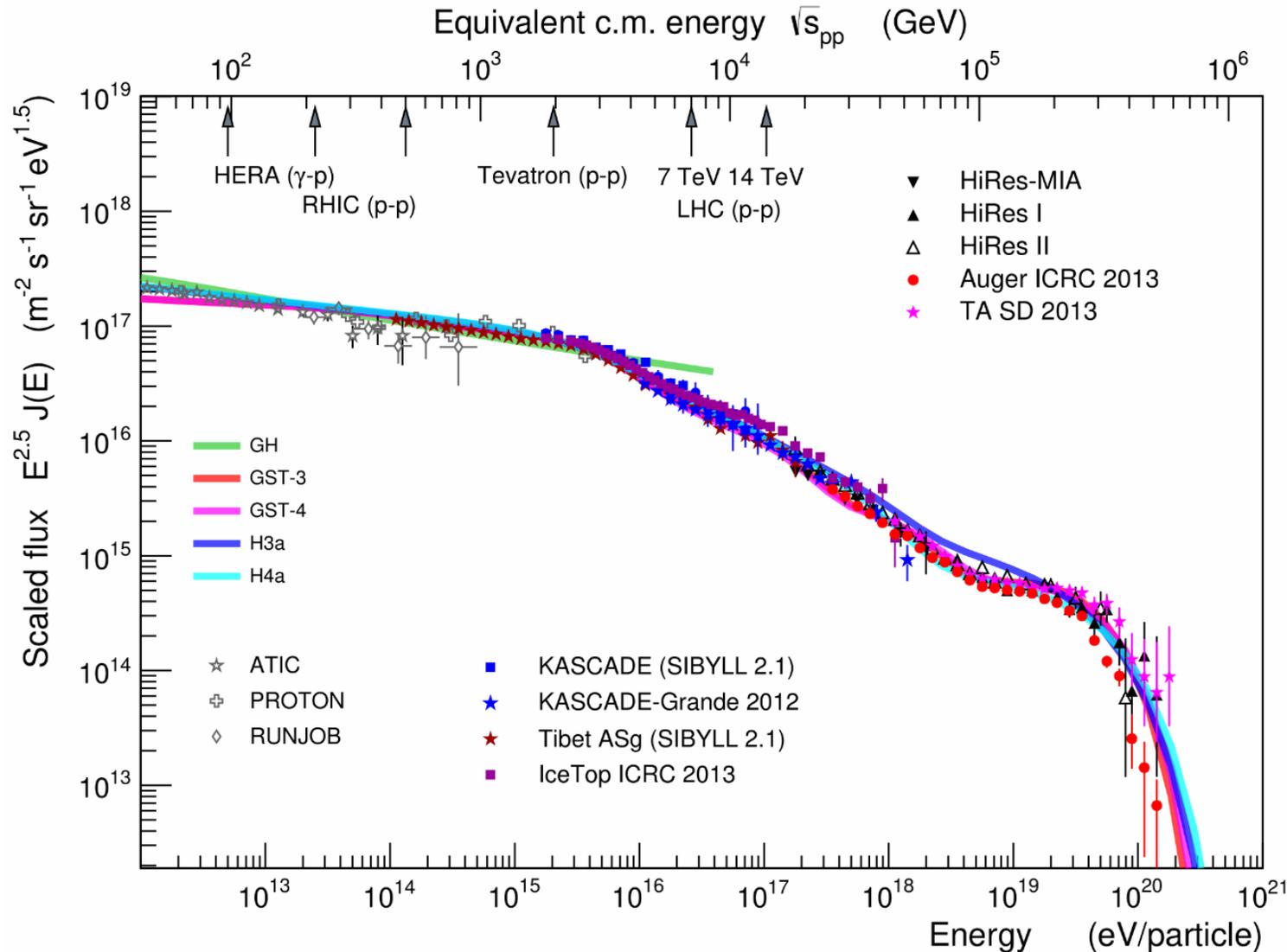
100 PeV p air shower



It's not 90% from first interaction
(as often assumed)



Comic ray spectrum at the top of the atmosphere



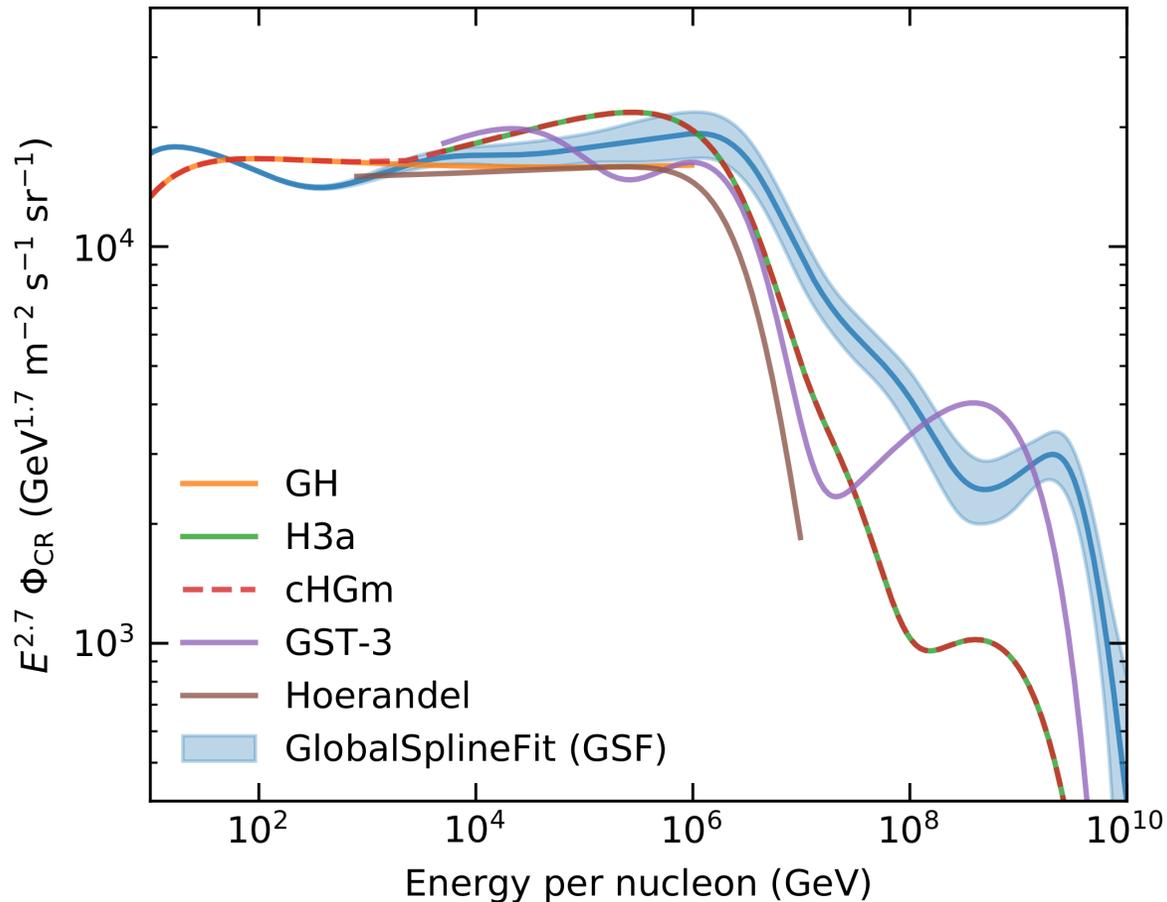
- None of the shown models is a real fit (with errors and covariance matrix)
- GST-X and HXa are quite extreme assumptions for UHECR
- No error estimates!

HXa:
 T. K. Gaisser,
Astropart. Phys. 35 (2012) 801–806
 GH:
 T. K. Gaisser and M. Honda,
Ann. Rev. Nucl. Part. Sci. 52 (2002) 153–199
 GST-X:
 T. K. Gaisser, T. Stanev, and S. Tilav,
Front. Phys.(Beijing) 8 (2013) 748–758

Update of GH (not shown):
 J. Evans, D. Porzio et al., 1612.03219



New GSF all-nucleon flux (input for atm. neutrino calculations)



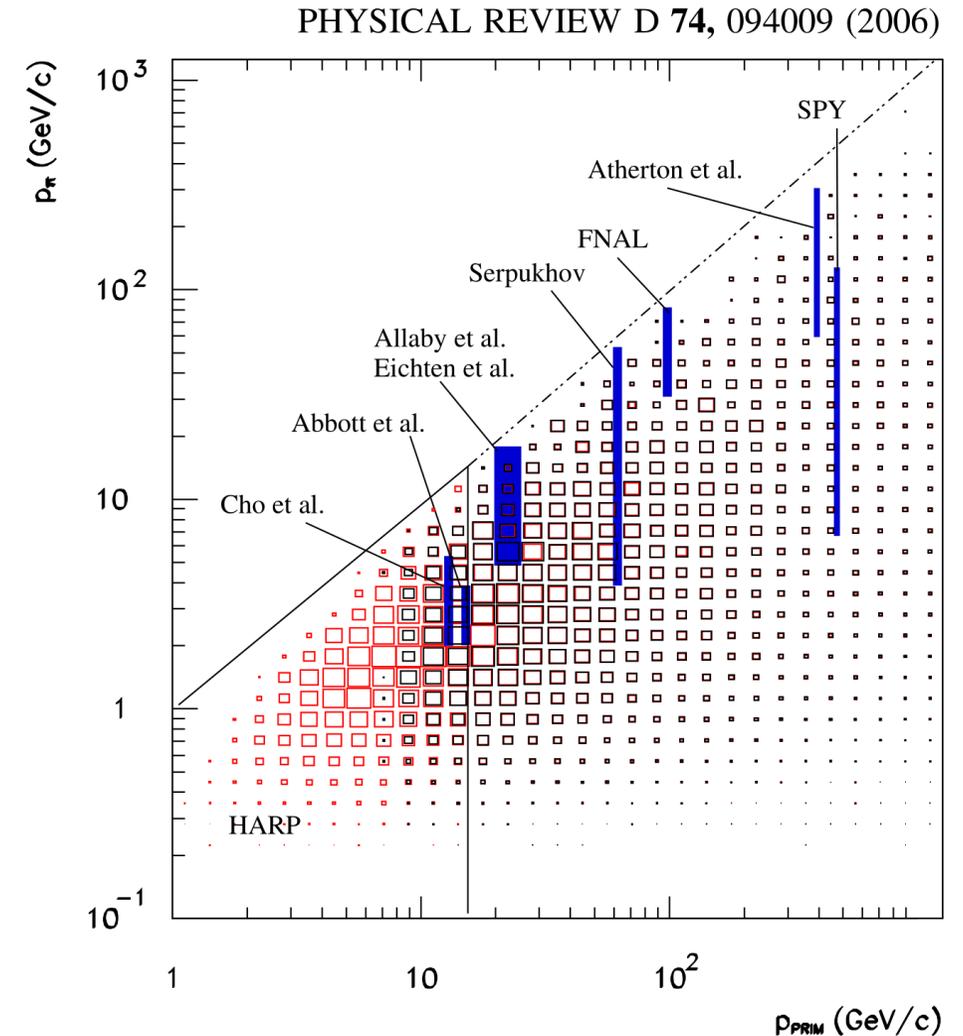
- Novel global fit to entire CR spectrum: *Dembinski et al., PoS(ICRC2017) 533*
- Wavy feature at lower energies are due to the hardening of proton and He spectra
- Increase of error around 10 TeV because of the gap between direct and indirect exp.
- Higher flux at the knee and harder spectral index between knee and ankle
- Latter effect comes from the **lighter composition at the knee** as in other models
- Mainly driven by KASKADE Gr. and latest data from IceTop and TUNKA



Hadronic uncertainties: current state of the art

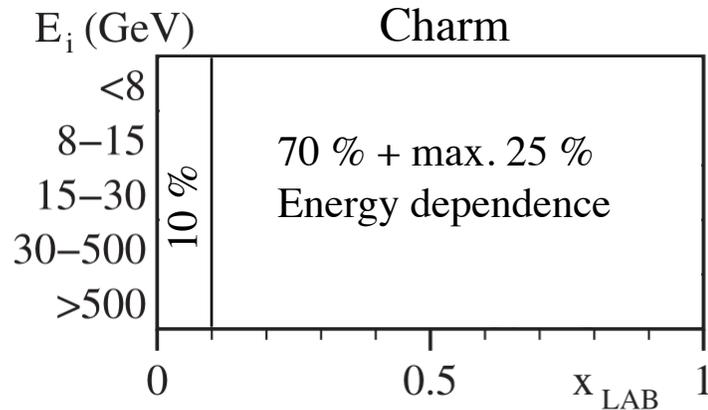
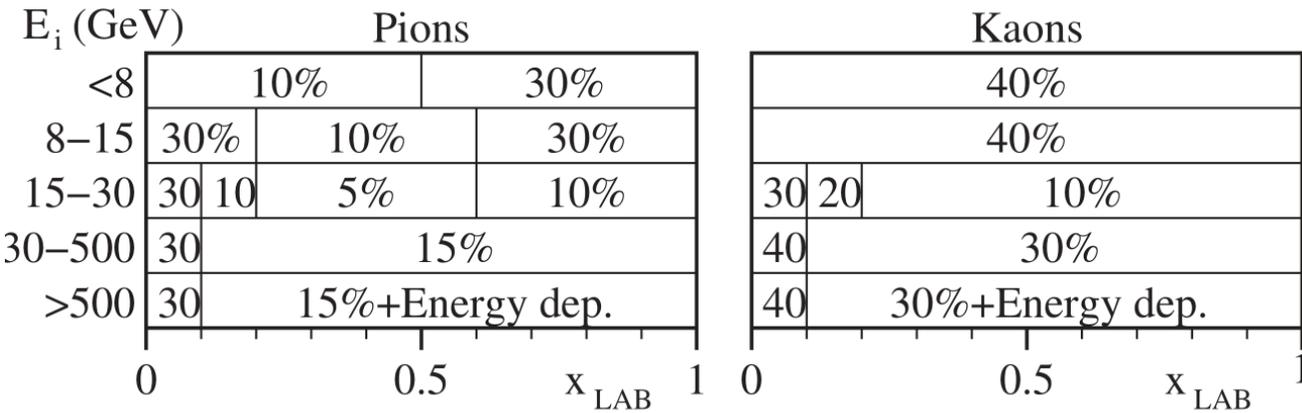
- > “*Uncertainties in atmospheric neutrino fluxes*”, G. D. Barr, S. Robbins, T. K. Gaisser, and T. Stanev, Phys. Rev. D 74, 094009 (2006)
- > Cut phase-space in regions/slices in E_{lab} and x_{lab} and **assign** uncertainty to each slice (uncorrelated)
- > **Draw-back 1:** Uncertainty assigned by hand and judged only from availability of experimental data (not how well TARGET described it)
- > **Draw-back 2:** The “central value” is assumed to be TARGET. Scheme doesn’t tell anything about “best estimate”.

Up to 10 TeV neutrino energy, no muons.



Implementation of the “Barr scheme” in MCEq

The regions



- > Compute partial derivatives wrt. phase-space regions, i.e. $\frac{\partial \Phi_\nu}{\partial W}$
- > No correlations between phase-space regions (as in Barr et al.) or add. correlations

Elements of Jacobian (numerical)

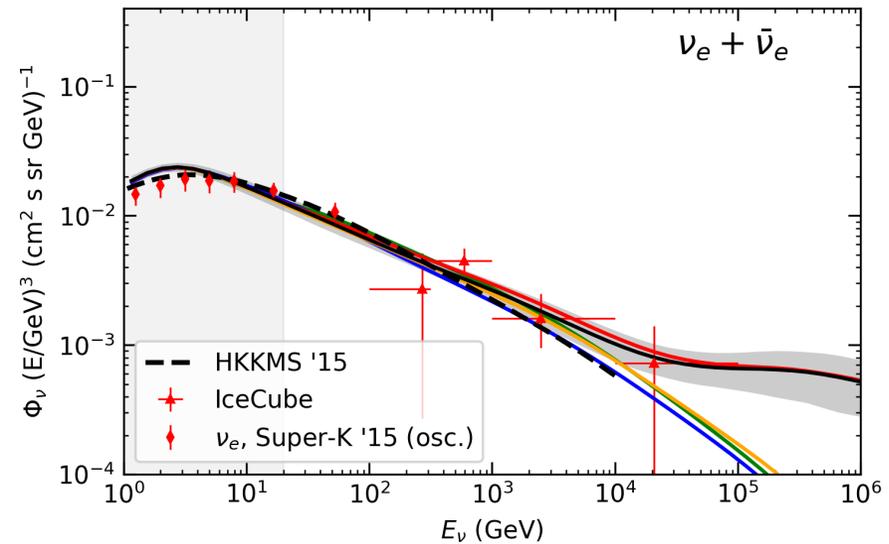
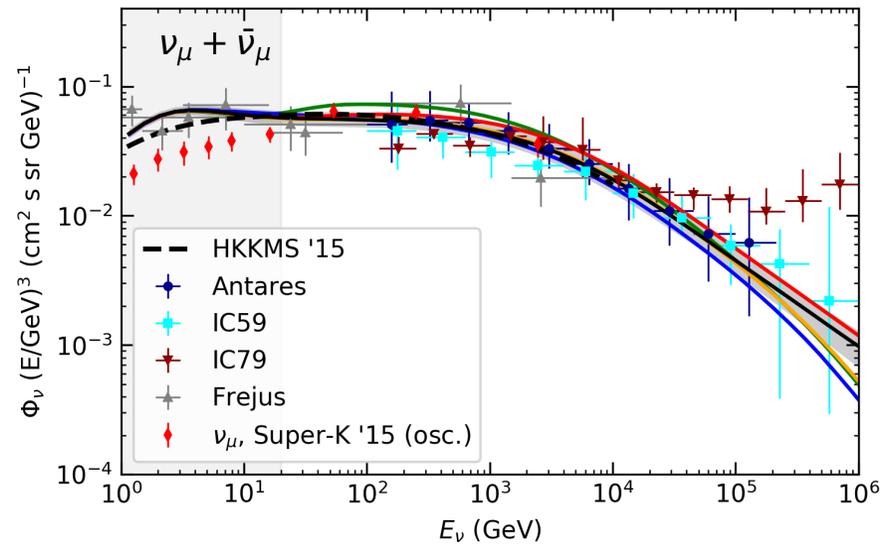
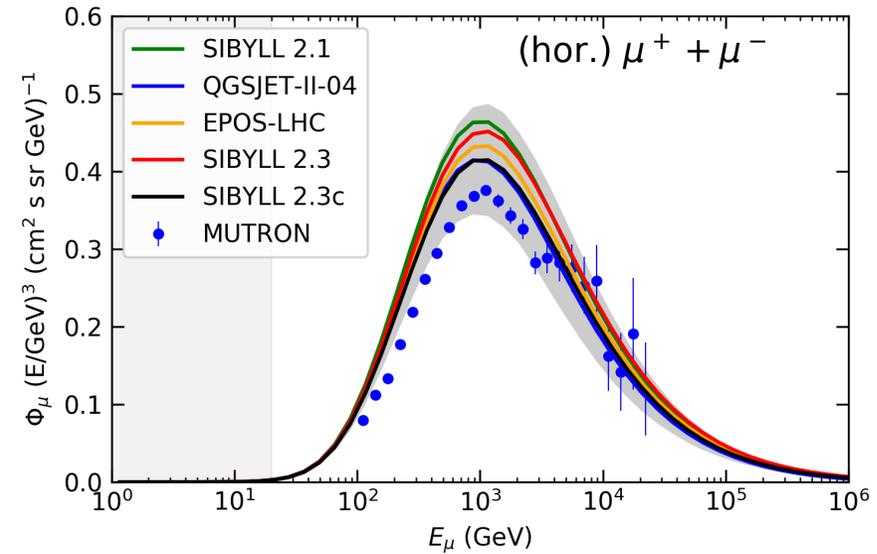
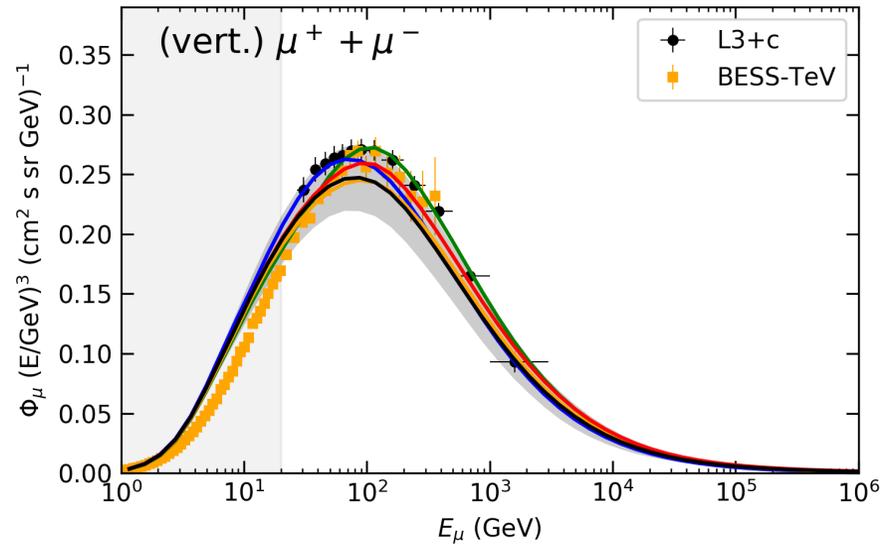
$$J_{E_i j} = \frac{\partial \Phi_\nu(E_i)}{\partial p_j} = \frac{\Phi_\nu(\delta p_j+) - \Phi_\nu(\delta p_j-)}{2\delta p_j}$$

Error propagation

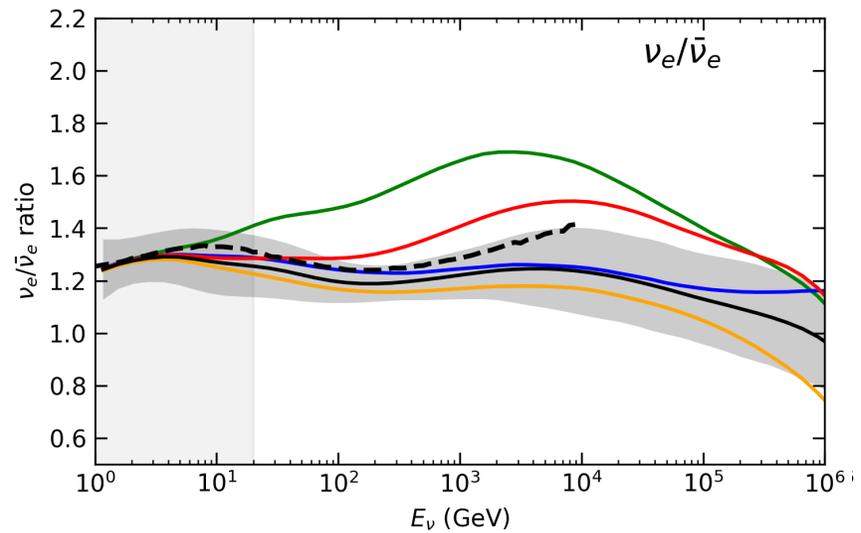
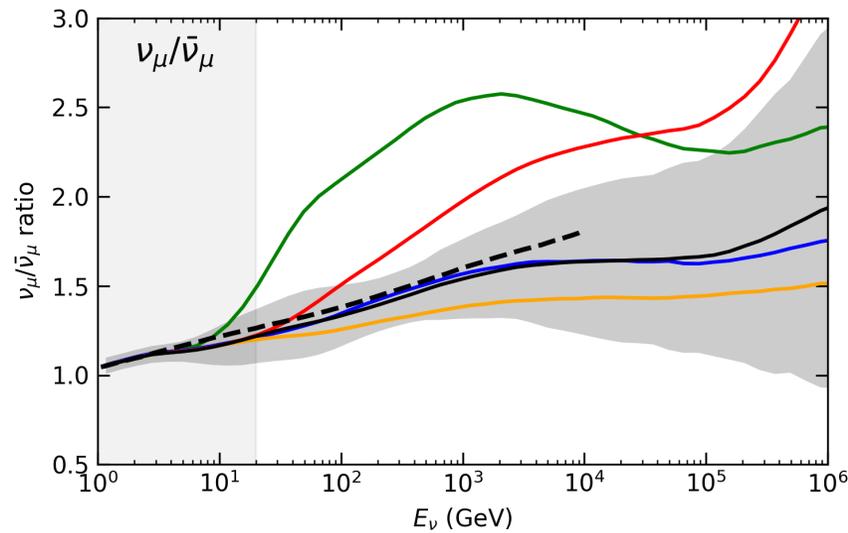
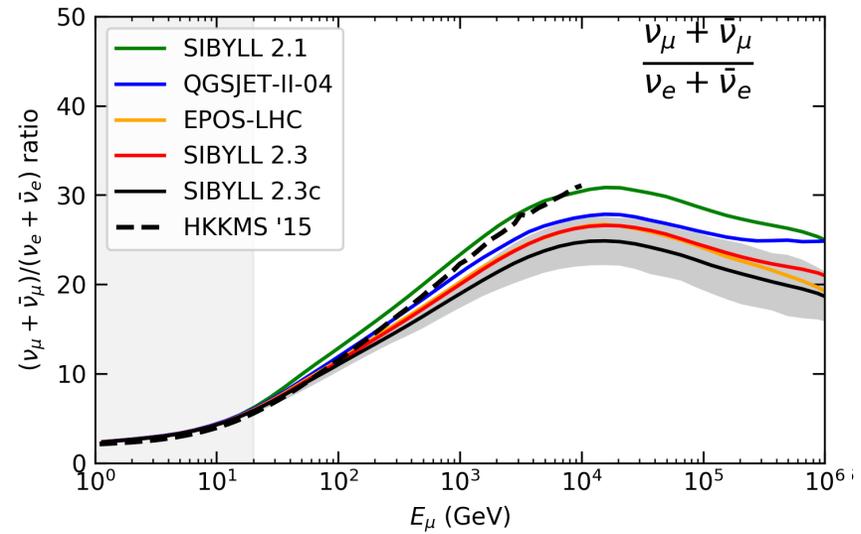
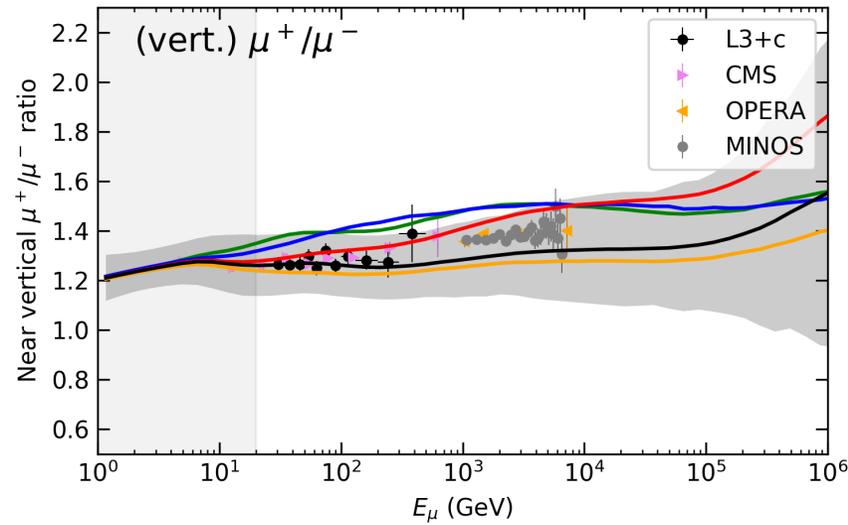
$$\text{cov}[\Phi_\nu(E_i), \Phi_\nu(E_j)] = \sum_{mn} J_{E_i m} J_{E_j n} \text{cov}[p_m, p_l]$$



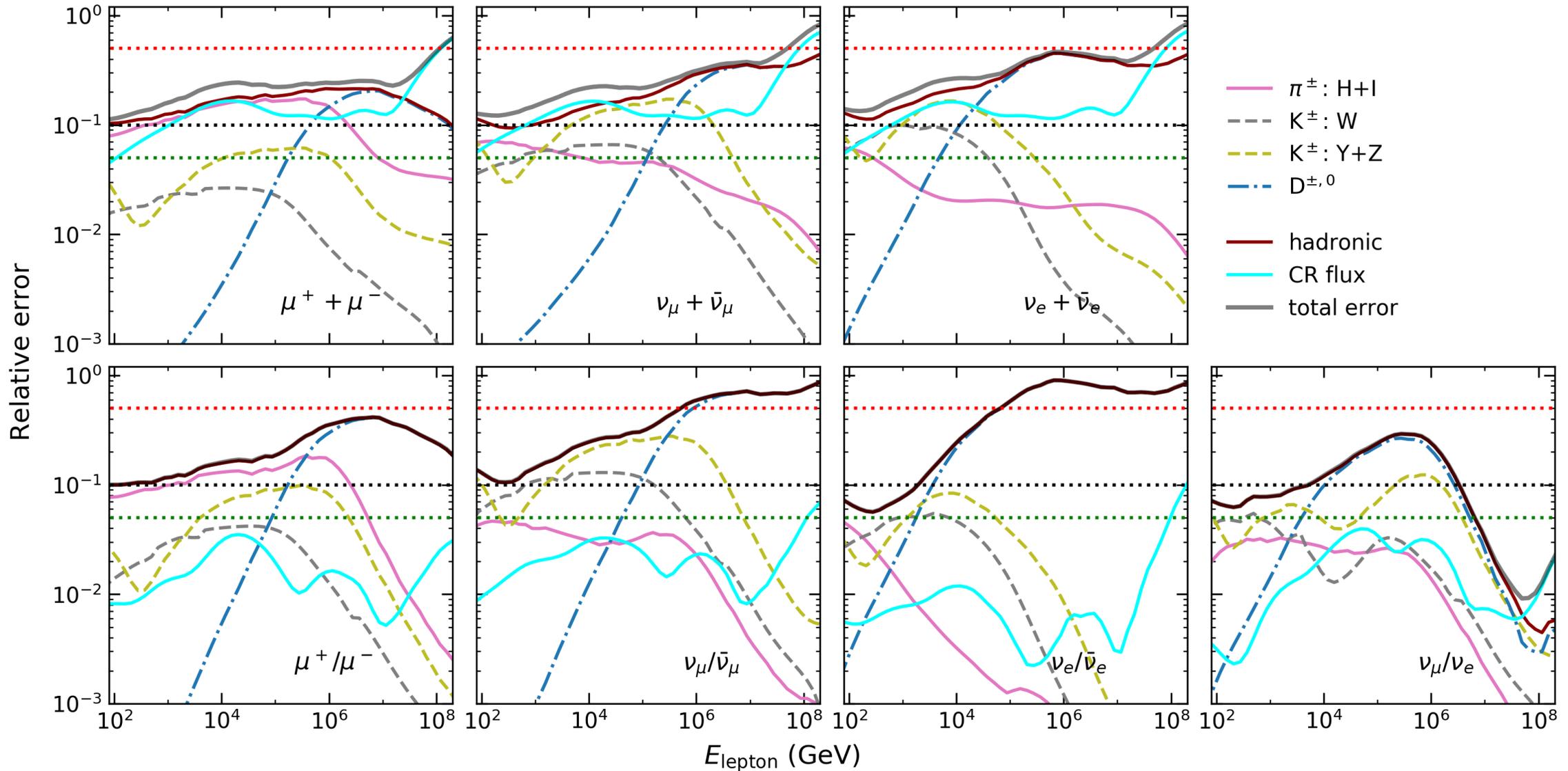
Uncertainties of lepton fluxes



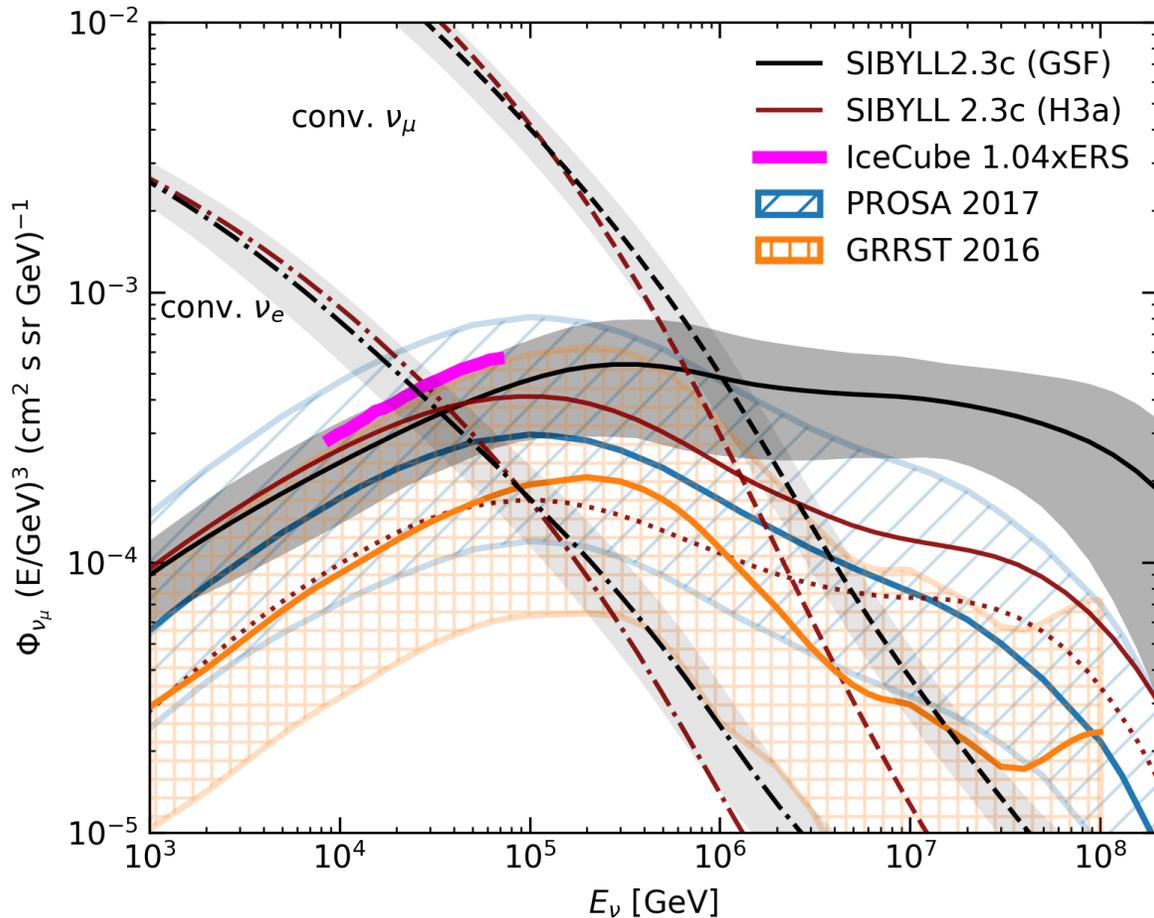
Uncertainties of lepton ratios



Hadronic uncertainties for high energy leptons



Prompt neutrinos from decays of charmed mesons



- SIBYLL 2.3c is the only full MC model
- Compatible with LHC data and IceCube limit
- New CR flux model (GSF) changes situation a bit
- Uncertainties from QCD very large and calculations are compatible
- Uncertainties from pQCD calculations are presumably overestimated, since LHC measurements of charm are not taken into account

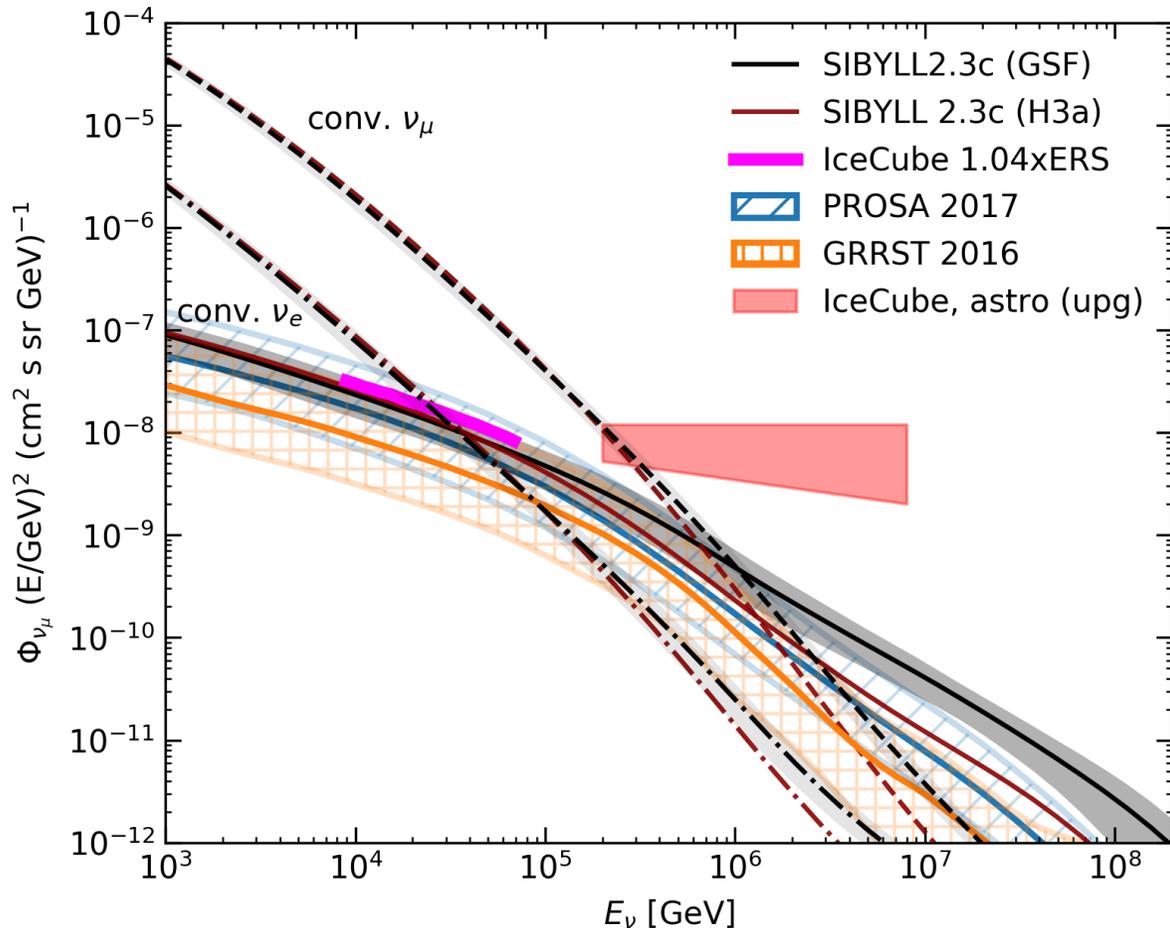
IceCube: *Astrophys.J.* 833 (2016)

GMS: Garzelli et al., *JHEP* 1510 (2015) 115

BERSS: Bhattacharya et al. *JHEP* 2015: 110



Prompt neutrinos from decays of charmed mesons



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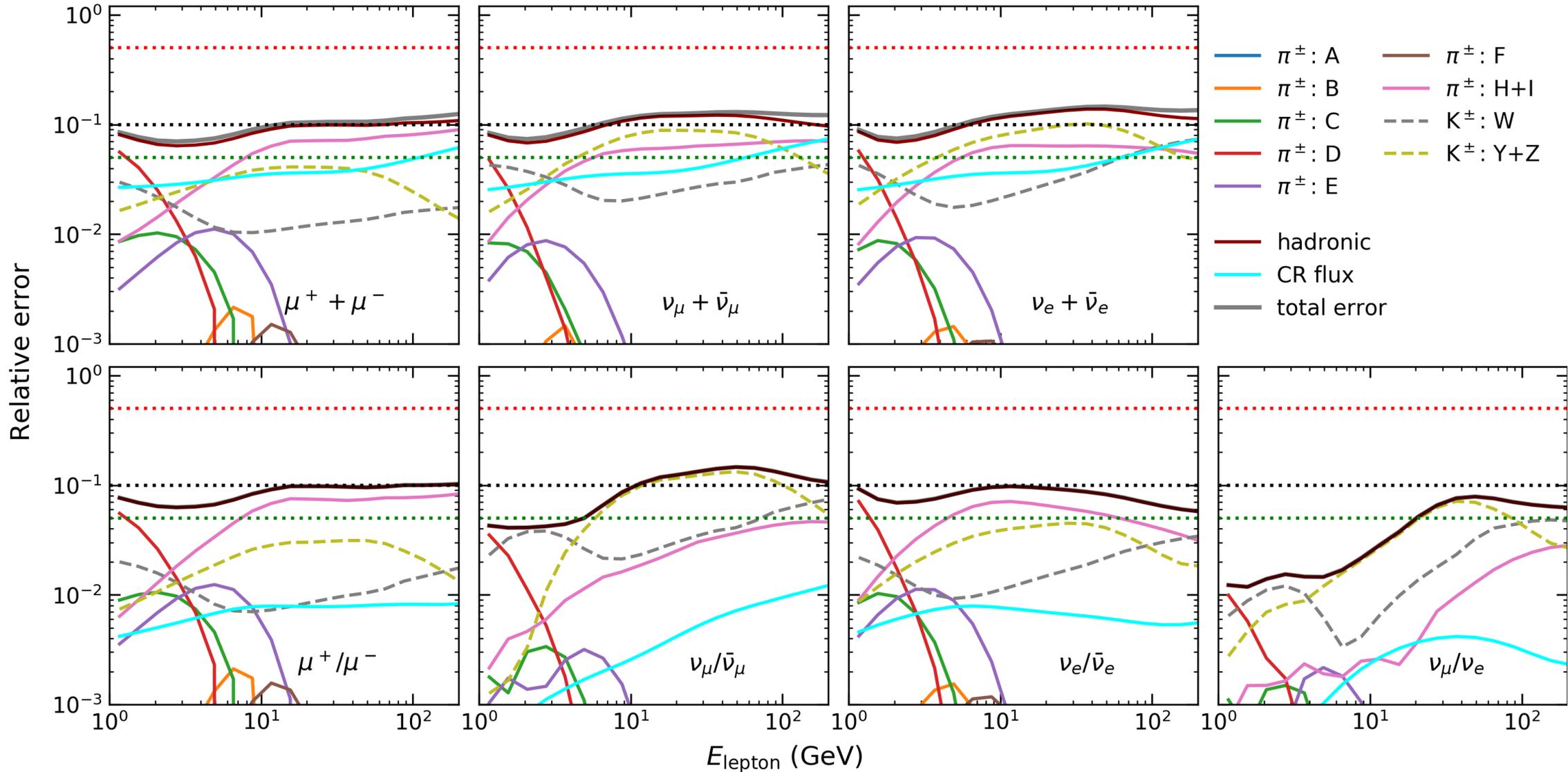


Conclusions

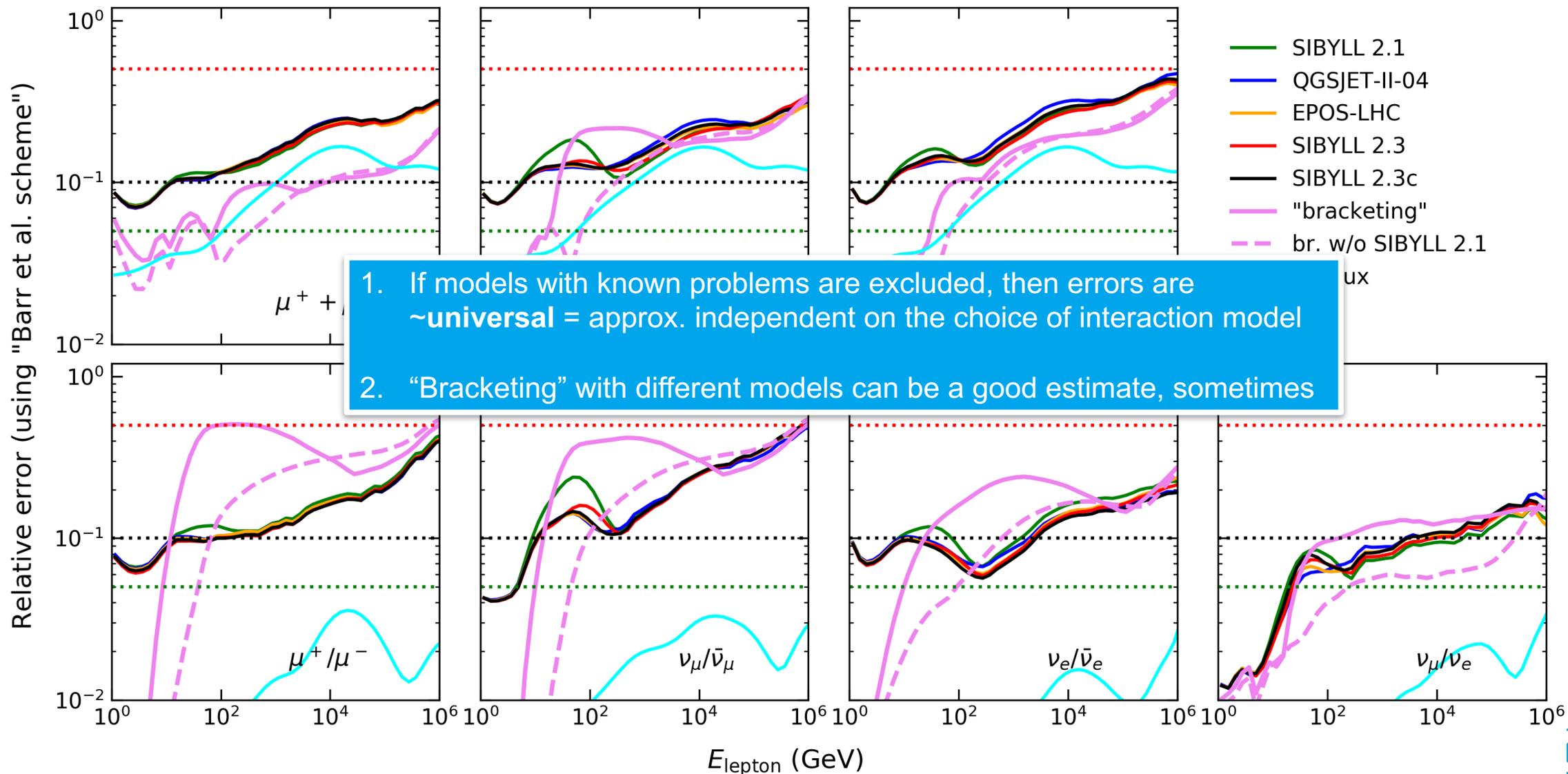
- > Relevant cosmic ray energies, contributing particles and their phase-space understood in detail.
- > Progress transparent to the community through an open source code, support, etc.
- > Cosmic ray uncertainties are quantified to our best knowledge. The numerical model including the covariance matrix will be open source, as well.
- > Uncertainties, at the level of Barr et al. can be reproduced with MCEq and corresponding tools will be published, asap.
- > High-quality atmospheric muon and neutrino data (incl. systematics, error correlation matrix, etc.) would definitely help



Hadronic uncertainties below 200 GeV

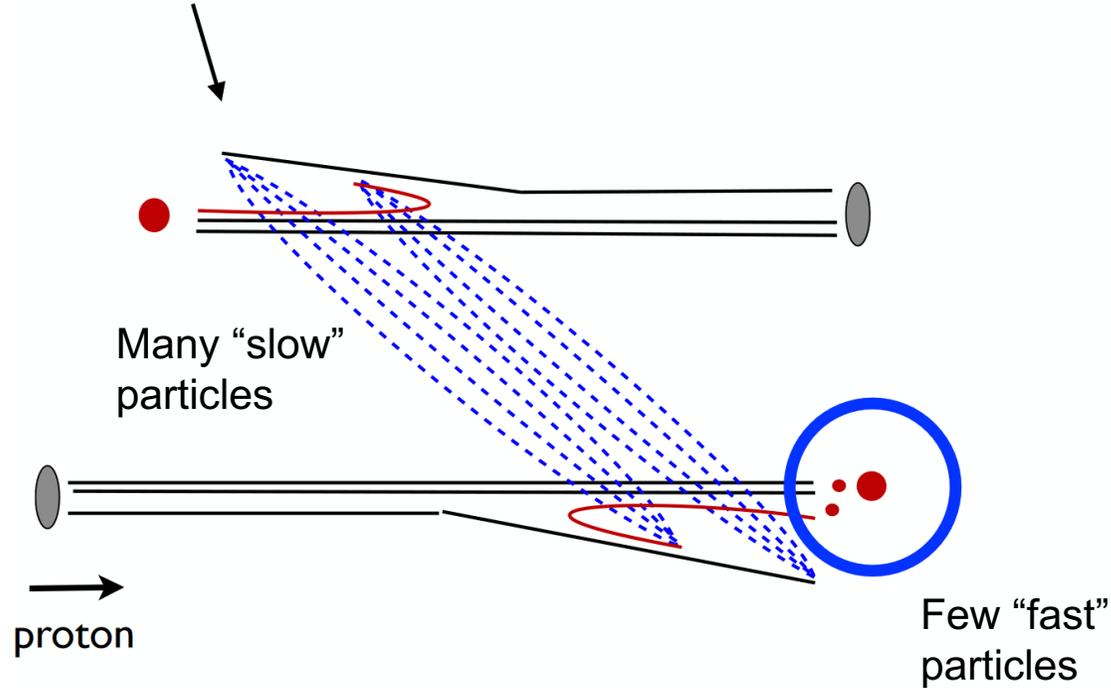


Hadronic uncertainties computed with different interaction models



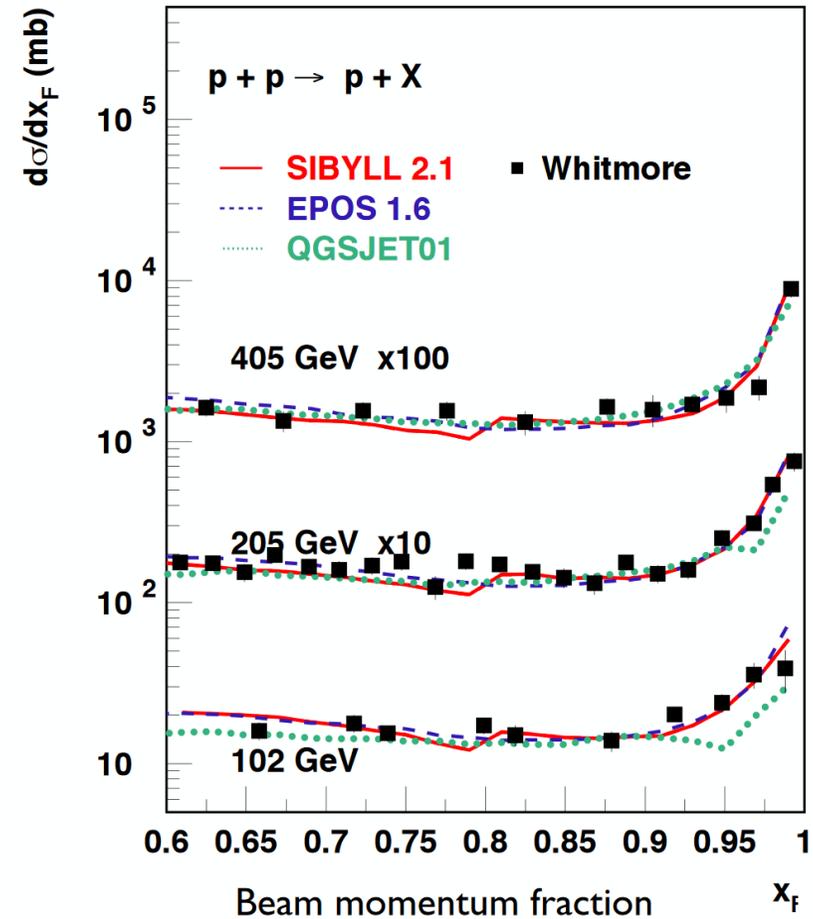
Leading particles

Model-dependent distributions of momentum given to partons

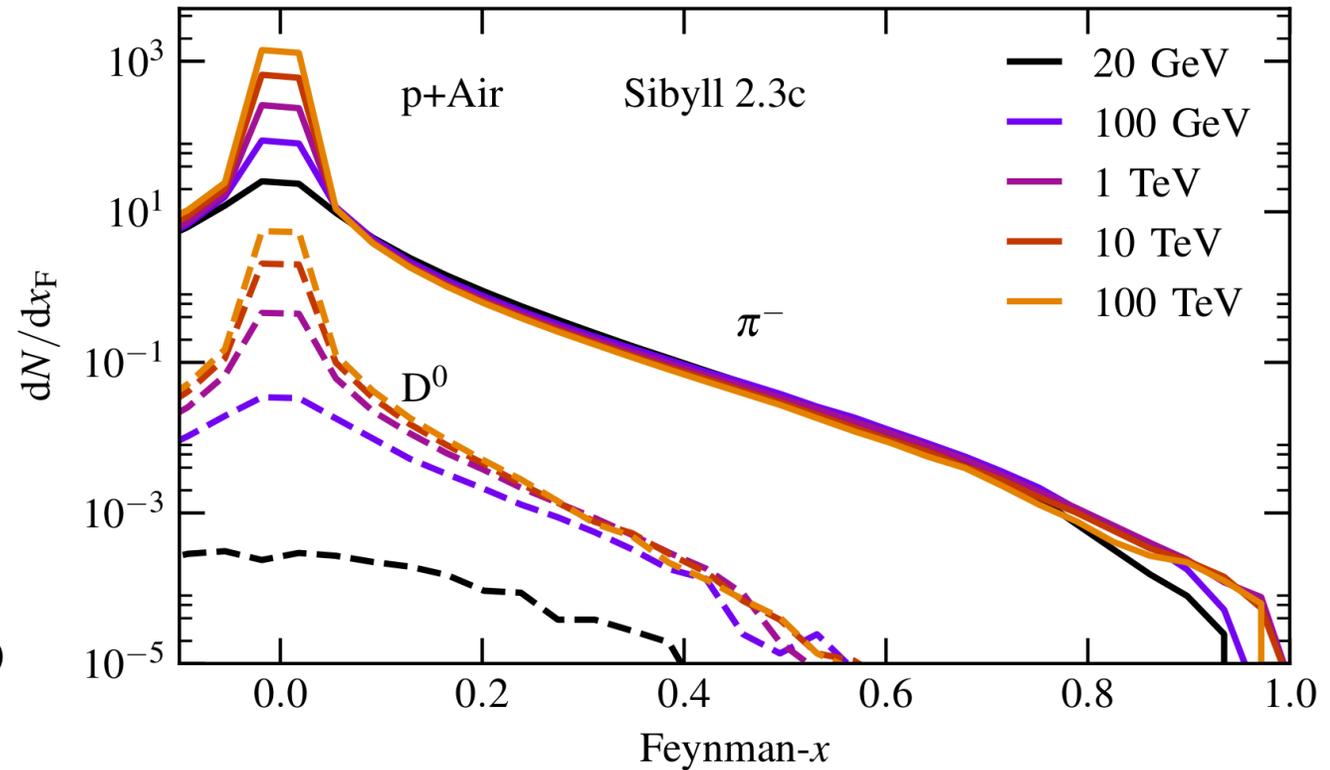
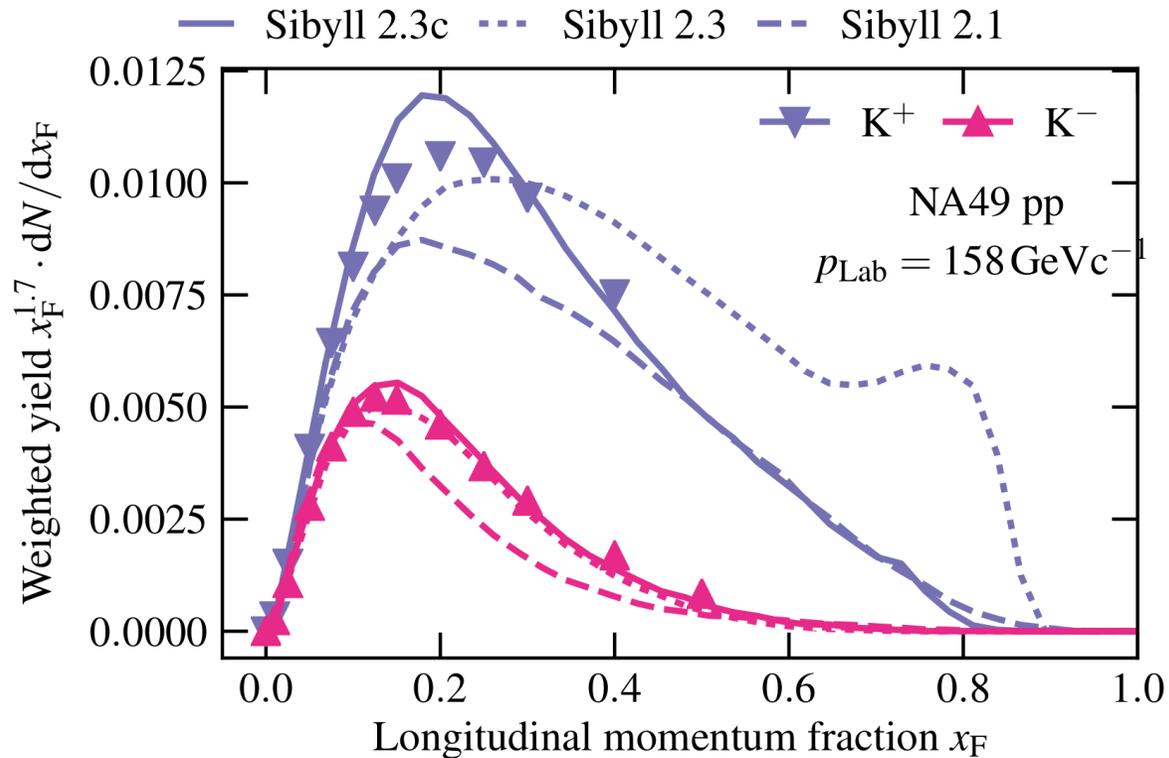


Fluctuations: Generation of sea quark anti-quark pair and leading/excited hadron

Leading particle effect



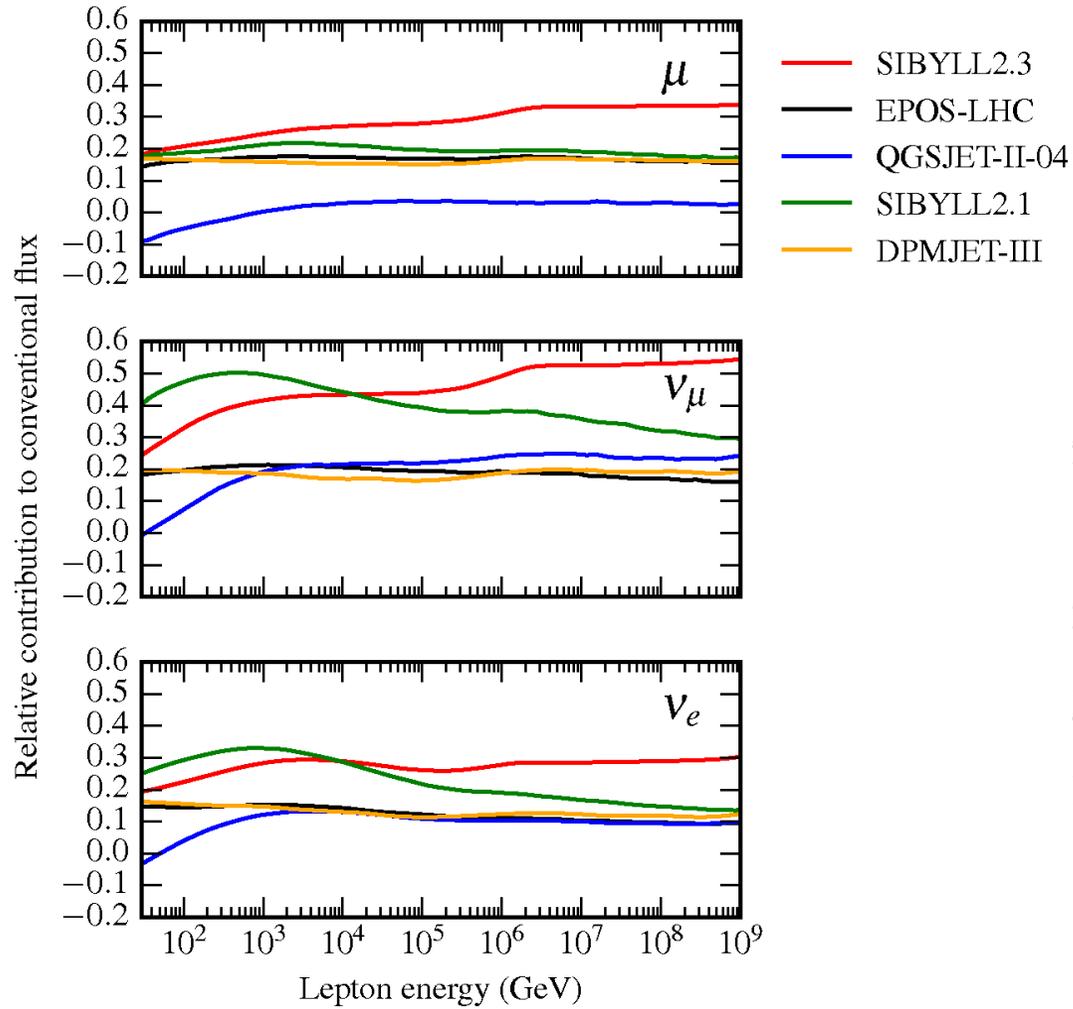
Main changes in SIBYLL 2.3c are related to scaling and leading Kaons



- > Problems in SIBYLL 2.3 related to remnant excitation model
- > No changes to charm in 2.3c
- > Small changes for air-shower physics

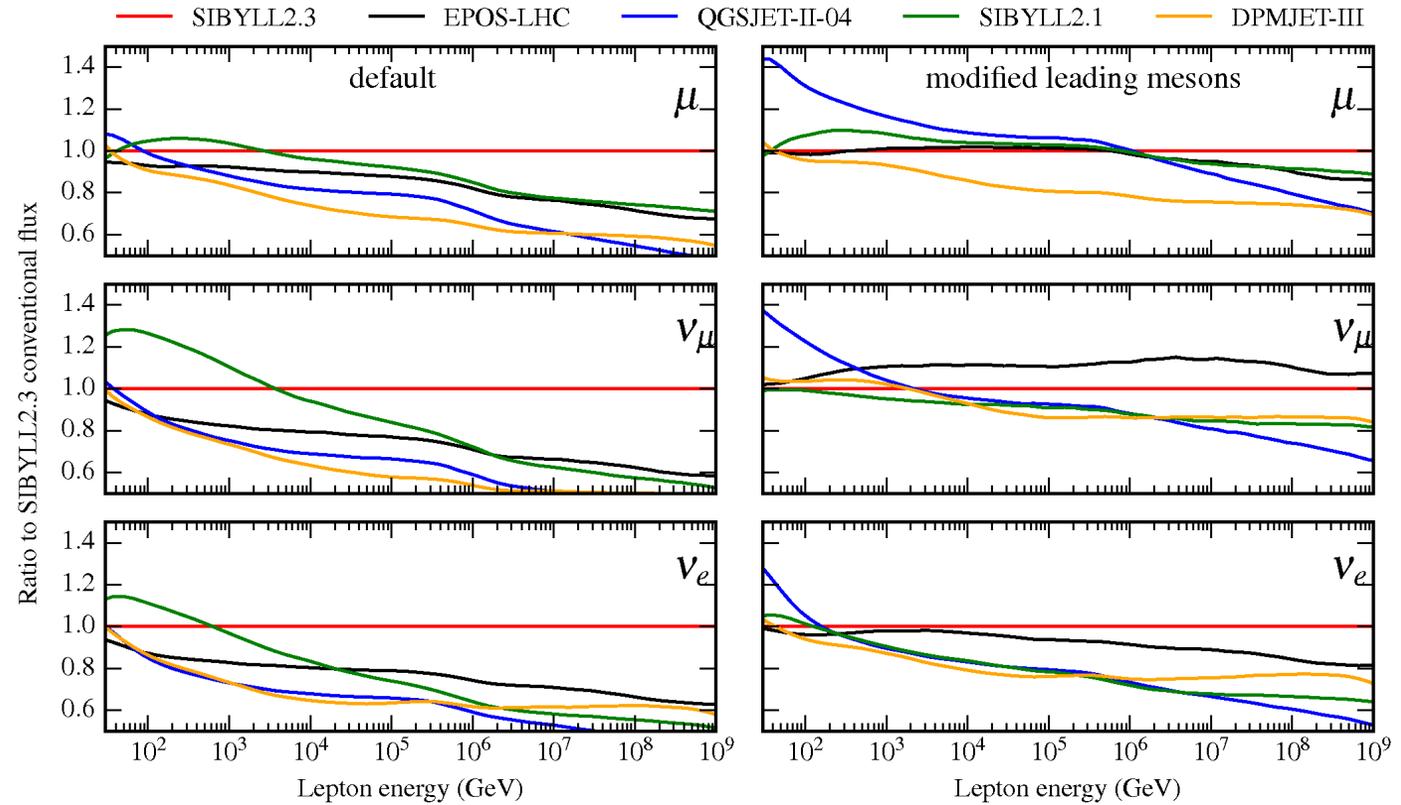


Contribution of the leading particle effect



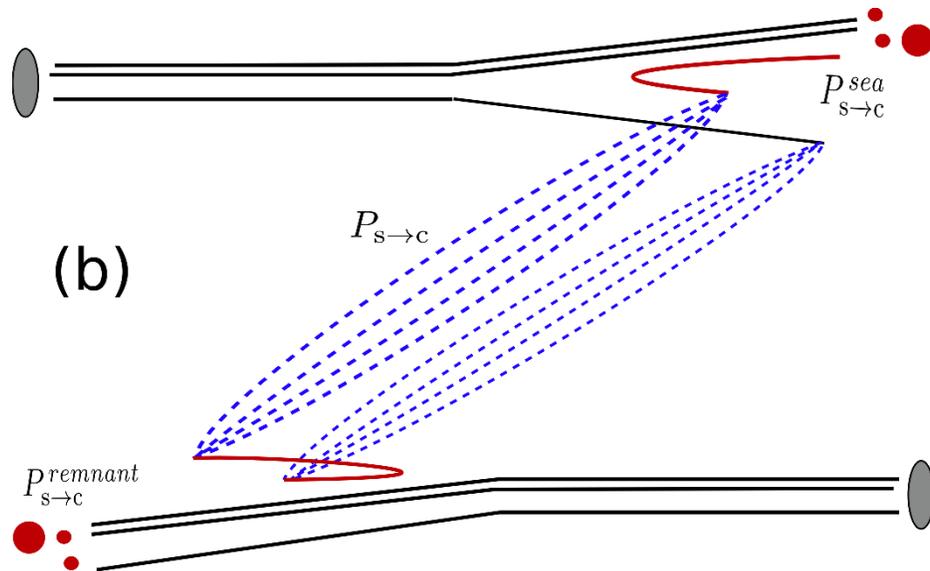
Larger effect in SIBYLL compared to other models

Modeling of leading particle effect contributes to large uncertainties

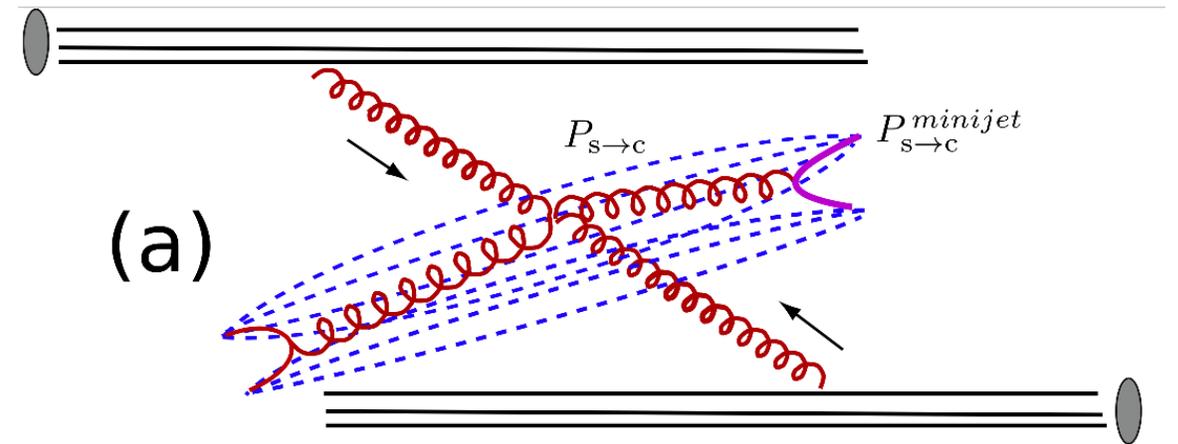


Charm in SIBYLL

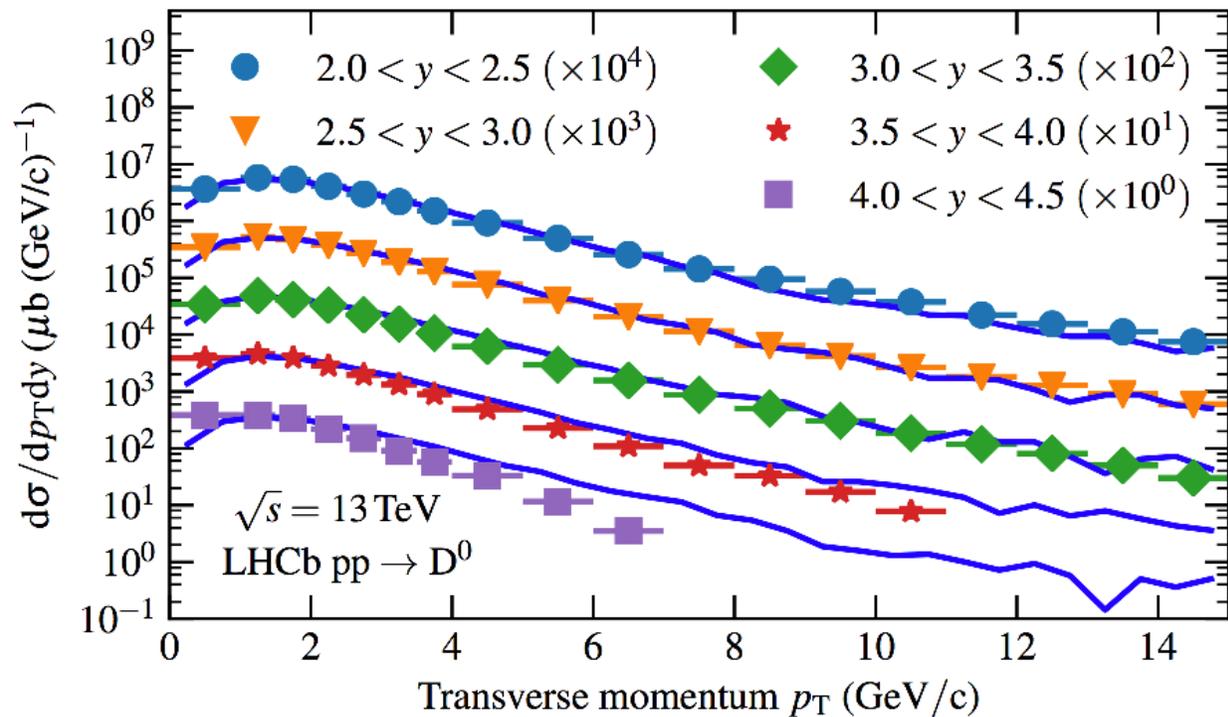
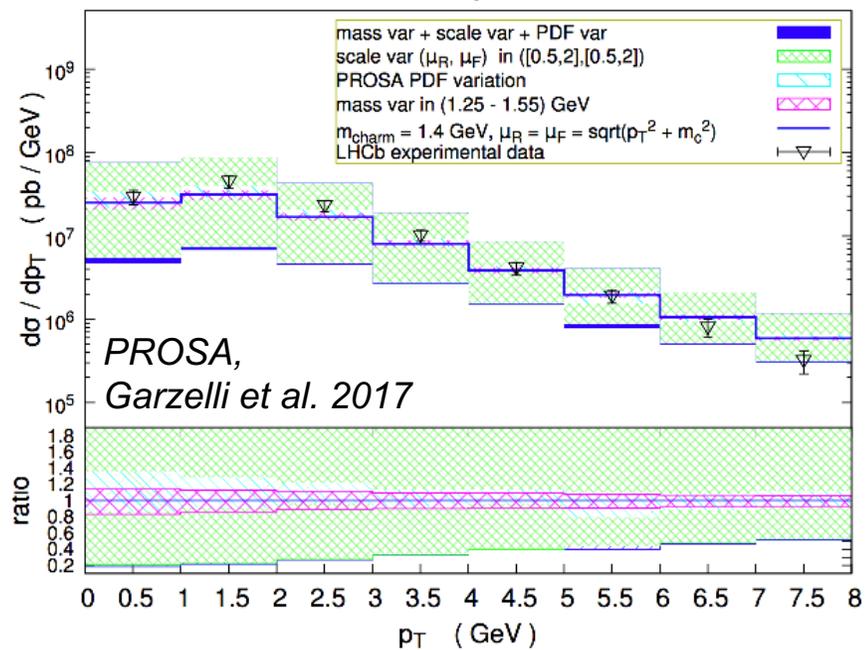
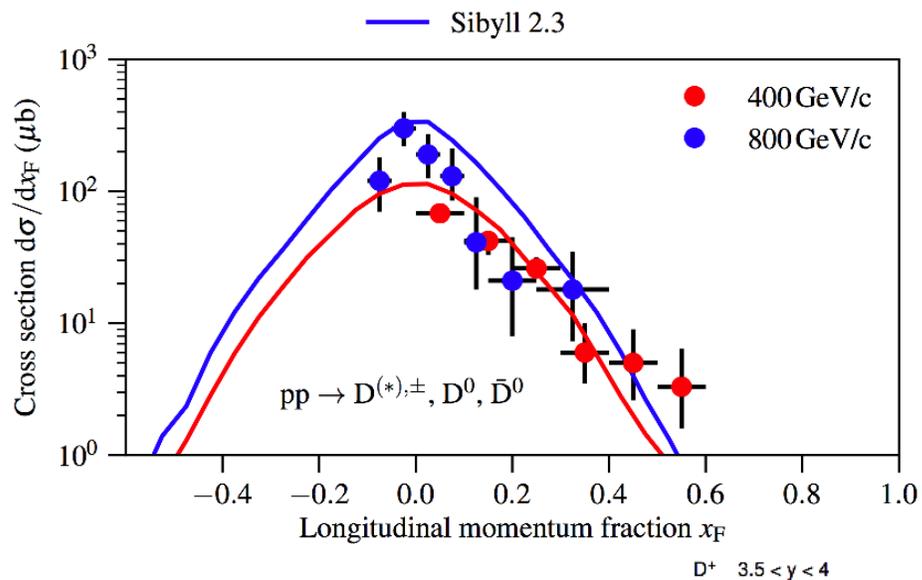
“non-perturbative/leading” charm



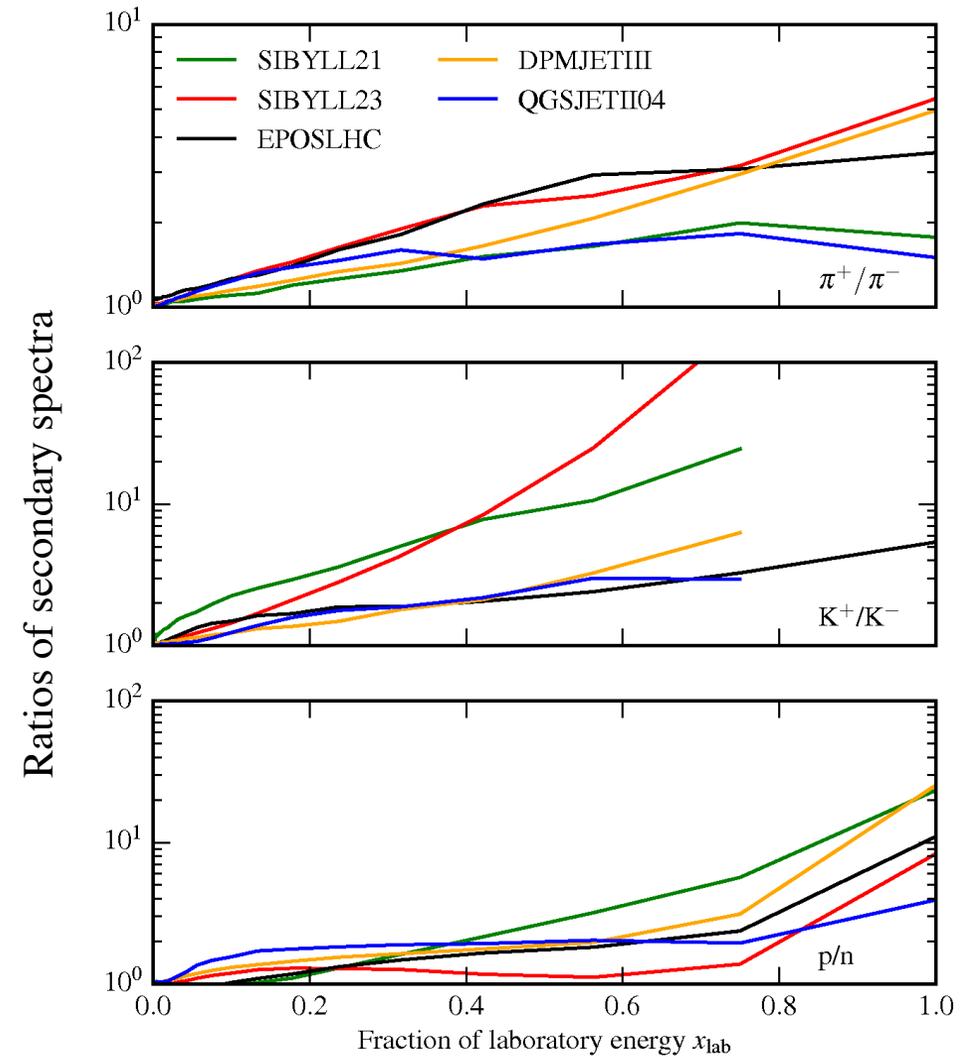
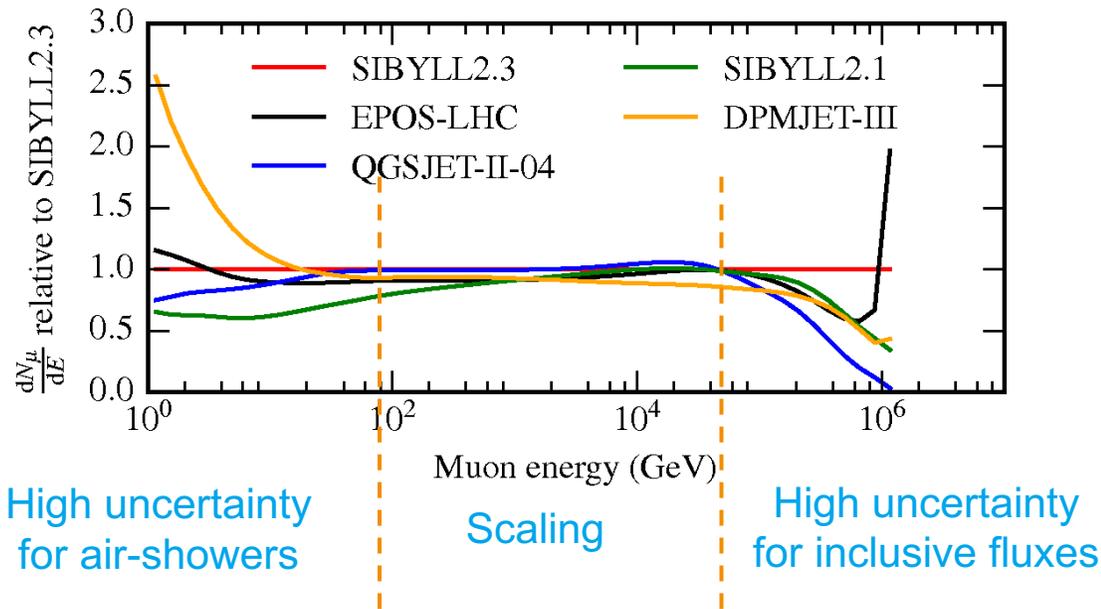
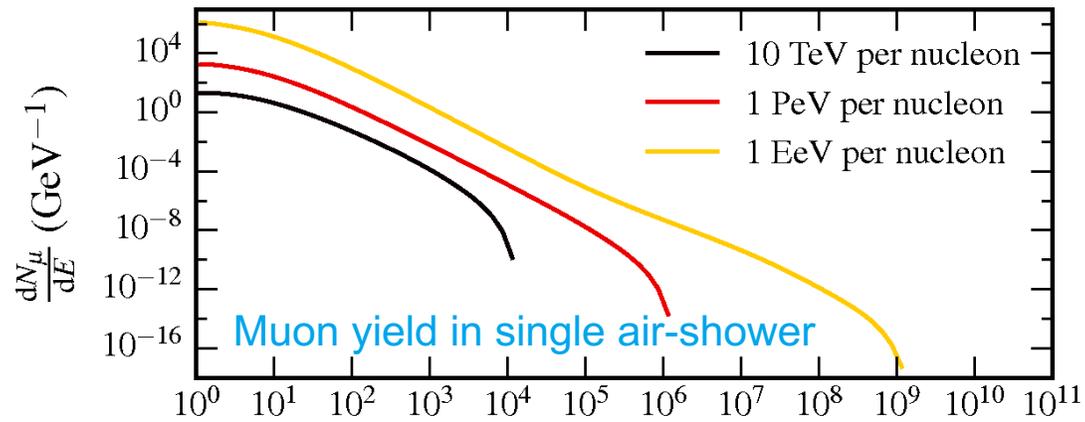
“pQCD/minijet” charm



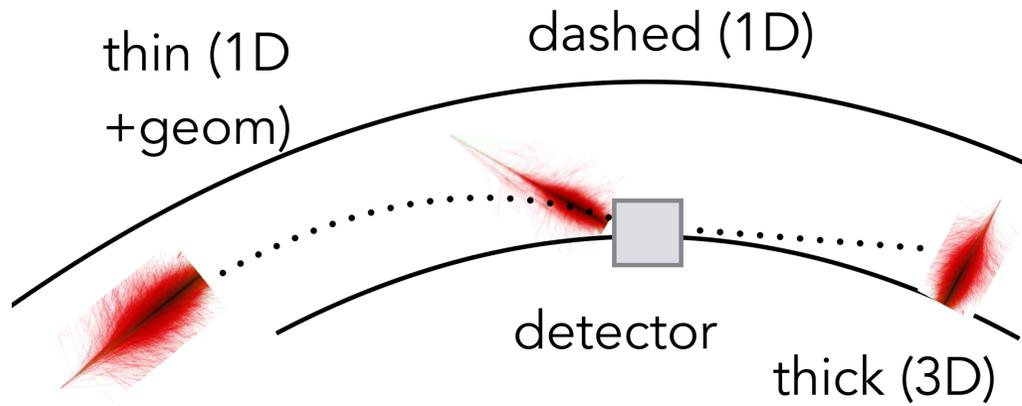
Charm in SIBYLL



Large uncertainties among models

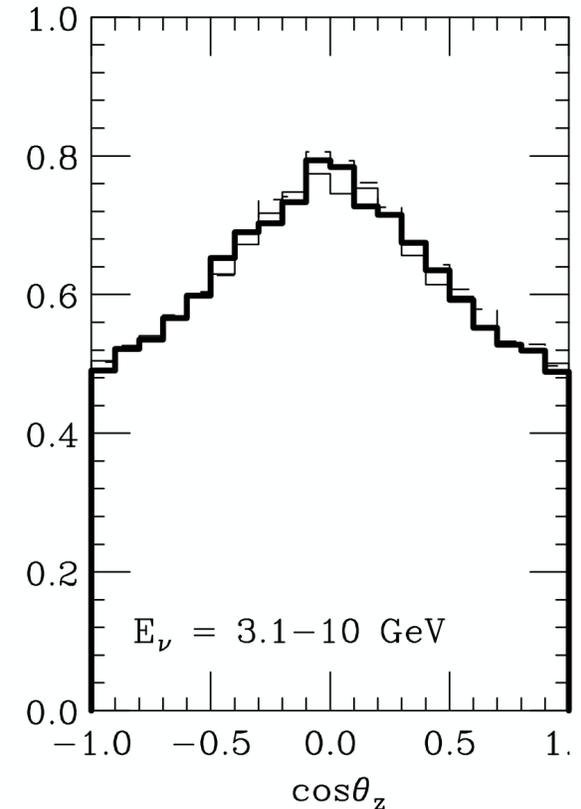
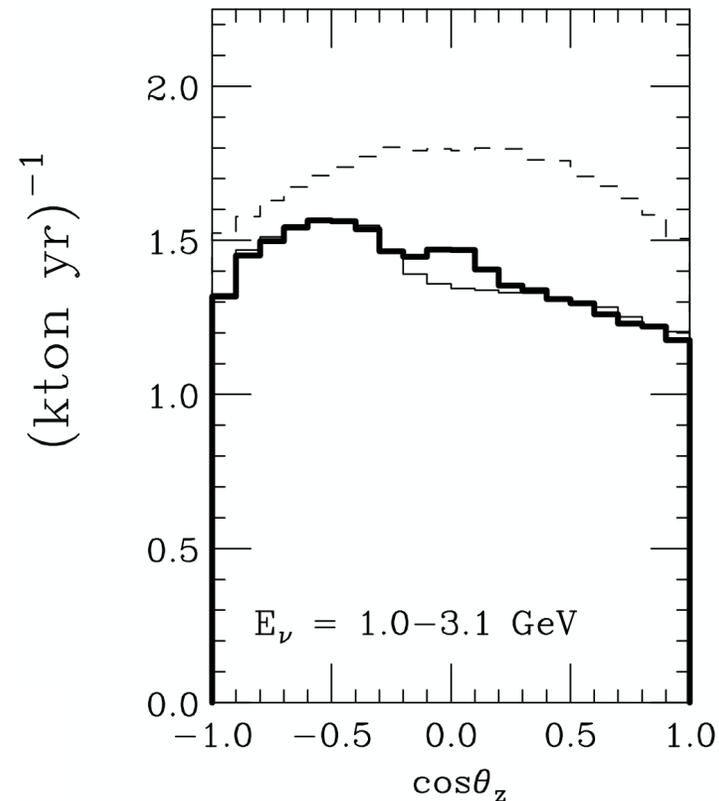


A few words on 3D calculations



A subset of 3D calculations

- [1] G. Barr, P. Lipari, S. Robbins, and T. Stanev, *International Cosmic Ray Conference 3*, 1411 (2003).
- [2] M. Honda, T. Kajita, K. Kasahara, and S. Midorikawa, *Phys. Rev. D* 83, (2011).
- [3] M. Honda, T. Kajita, K. Kasahara, S. Midorikawa, and T. Sanuki, *Phys. Rev. D* 75, (2007).
- [4] [1] G. Battistoni, A. Ferrari, P. Lipari, T. Montaruli, P. R. Sala, and T. Rancati, *Astroparticle Physics* **12**, 315 (1999).
- [5] J. Wentz, I. M. Brancus, A. Bercuci, D. Heck, J. Oehlschläger, H. Rebel, and B. Vulpescu, *Phys. Rev. D* 67, 073020 (2003).

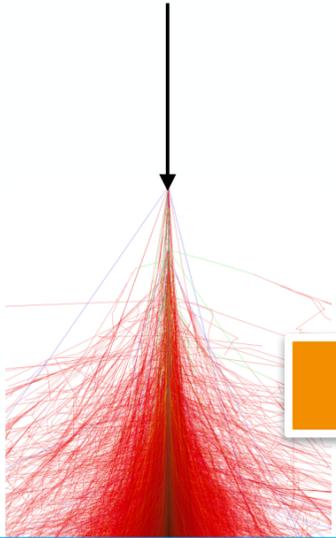


Classical solutions

1D MC calculation

For various "inputs"

primary spectrum,
zenith angle,
composition



Very slow

simulate "average"
air-shower

$$\Phi_{\mu}(E_{\mu}, \theta) = \sum_p \sum_{E_i} w_p(E_i) \times \mathcal{Y}_{p \rightarrow \mu}(E_{\mu}, E_i, \theta)$$

nuclei/
composition
primary spectrum
from CORSIKA

$$w_p(E_i) = \frac{1}{N_p(E_i)} \int_{E_i}^{E_{i+1}} \Phi_p(E') dE', \quad N_p(E_i) = \text{NSHOW}$$

of particles in virtual
detector

M. Honda et al. 1995, Agrawal et al. 1995, Fedynitch et al. 2012

Semi-analytical

T. K. Gaisser, R. Engel and E. Resconi,
Cosmic Rays and Particle Physics, 2nd edition

$$\Phi_{\nu}(E) = \frac{\phi_N(E)}{1 - Z_{NN}} \left(\frac{A_{\pi\nu}}{1 + \mathcal{B}_{\pi\nu} E \cos \theta / \varepsilon_{\pi}} + \frac{A_{K\nu}}{1 + \mathcal{B}_{K\nu} E \cos \theta / \varepsilon_K} \right)$$

- > 3 – 6 |
- > De-co

Limited detail and
precision

- > Numerous assumptions about spectral shape, atmospheric model, etc.
- > Only asymptotical low- and high-energy solutions



MCEq: Matrix Cascade Equations

Matrix-form

$$\frac{d\Phi_{E_i}^h}{dX} = -\frac{\Phi_{E_i}^h}{\lambda_{\text{int},E_i}^h} - \frac{\Phi_{E_i}^h}{\lambda_{\text{dec},E_i}^h(X)} + \sum_{E_k \geq E_i}^{E_N} \sum_k \frac{c_k(E_k) \rightarrow h(E_i)}{\lambda_{\text{int},E_k}^k} \Phi_{E_k}^k \cdot \frac{\Phi_k(E_k, X)}{\lambda_{\text{int},k}(E_k)} + \sum_{E_k \geq E_i}^{E_N} \sum_k \frac{d_k(E_k) \rightarrow h(E_i)}{\lambda_{\text{dec},E_k}^k(X)} \Phi_{E_k}^k \cdot \frac{\Phi_k(E_k, X)}{\lambda_{\text{dec},k}(E_k, X)}$$

$$\frac{d}{dX} \phi = \left[(-\mathbf{1} + \mathbf{C}) \Lambda_{\text{int}} + \frac{1}{\rho(X)} (-\mathbf{1} + \mathbf{D}) \Lambda_{\text{dec}} \right] \phi.$$

\uparrow
geometry & atmosphere

flux vector

$$\vec{\phi} = \begin{pmatrix} \phi_p(E_0) \\ \phi_p(E_1) \\ \dots \\ \phi_p(E_N) \\ \phi_n(E_0) \\ \dots \\ \phi_n(E_N) \\ \phi_{\pi}^+(E_0) \\ \dots \\ \phi_{\bar{\nu}_e}(E_N) \end{pmatrix}$$



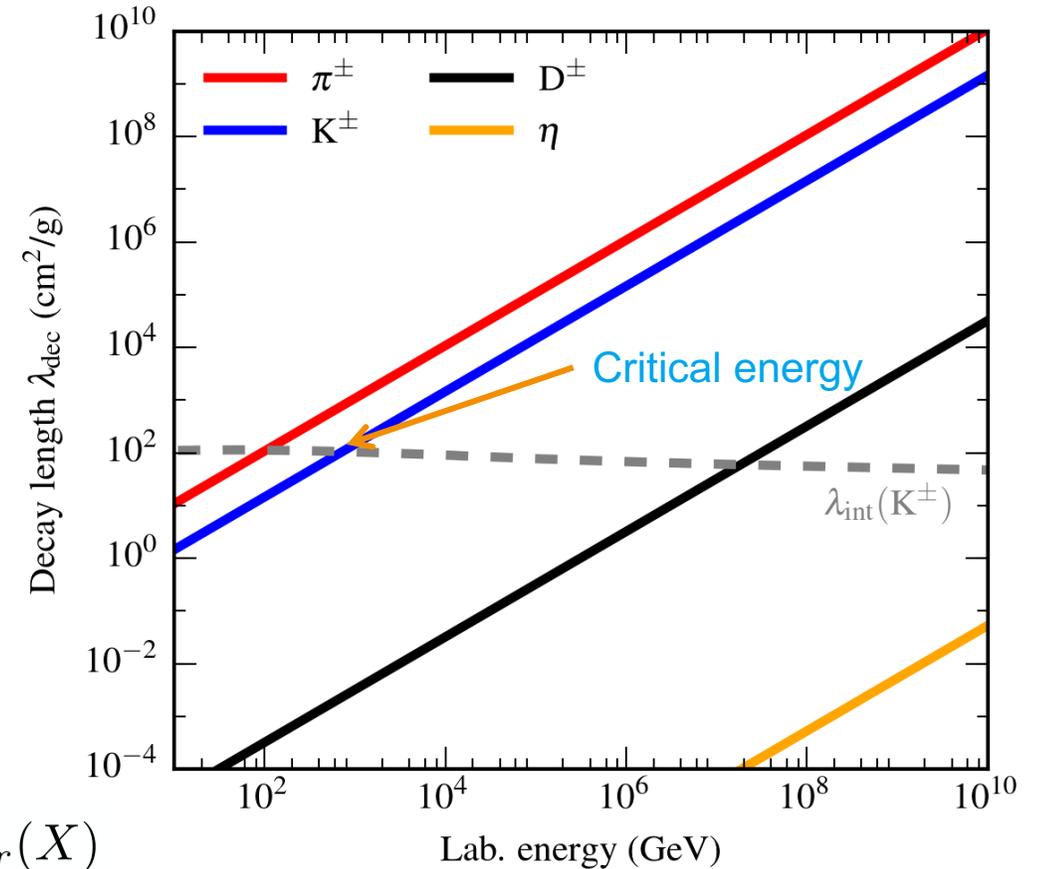
Competition between decay and interactions

- > Above critical energy, probability for interaction is higher than decay probability
- > Flux from one particular hadron species gets attenuated

particle	E_{crit} [GeV]
μ^\pm	1.0
π^\pm	115
K_L^0	205
K^\pm	850
K_S^0	1.2E+05
D^\pm	4.3E+07

$$\lambda_{dec,h}(E, X) = \frac{c\tau_h E \rho_{Air}(X)}{m_h c^2}$$

$$= \frac{EX \cos \theta}{E_{crit,h}}$$



Hadrons in MCEq

Leptons

$$\mu^+, \mu^-, \tau^+, \tau^-, \nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$$

Mesons

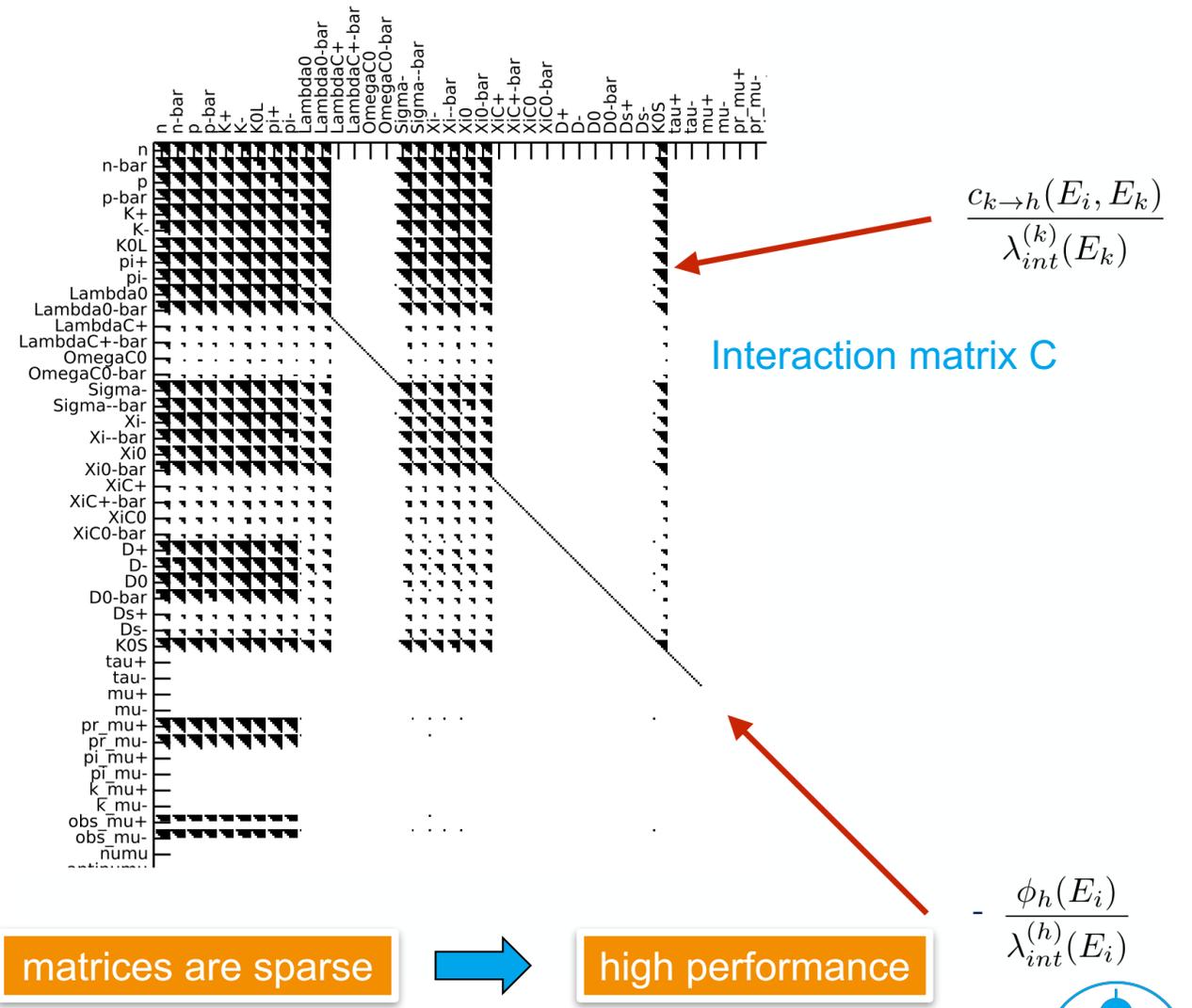
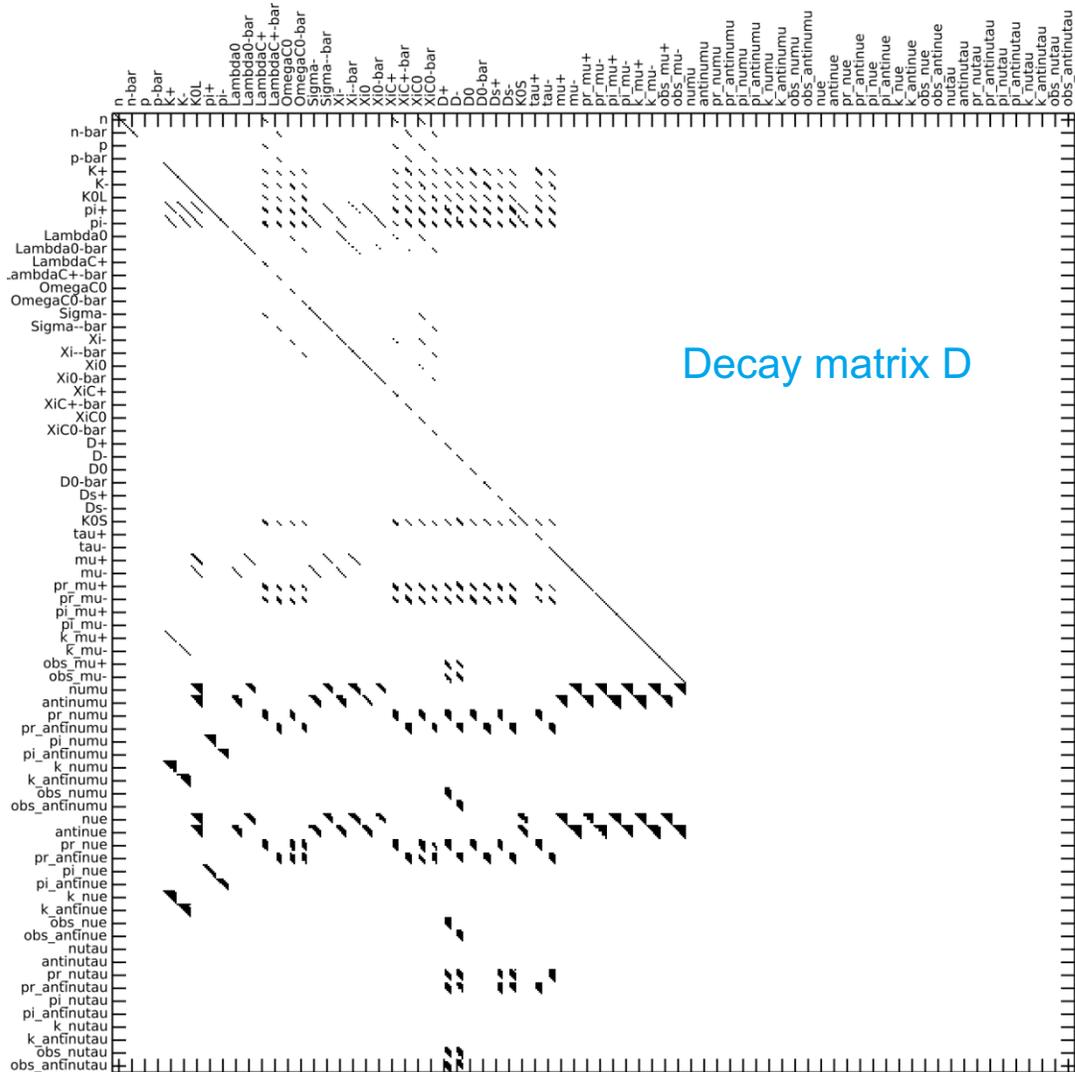
$$K^+, K^-, K_L^0, K_S^0, \pi^+, \pi^-, D^+, D^-, D^0, \bar{D}^0, D_s^+, D_s^-, K^{*+}, K^{*-}, K^{*0}, \bar{K}^{*0}, D^{*+}, D^{*-}, D^{*0}, \bar{D}^{*0}, \eta, \eta^*, \eta_C, J/\Psi, \omega, \phi, \pi^0, \rho^+, \rho^-, \rho^0$$

Baryons

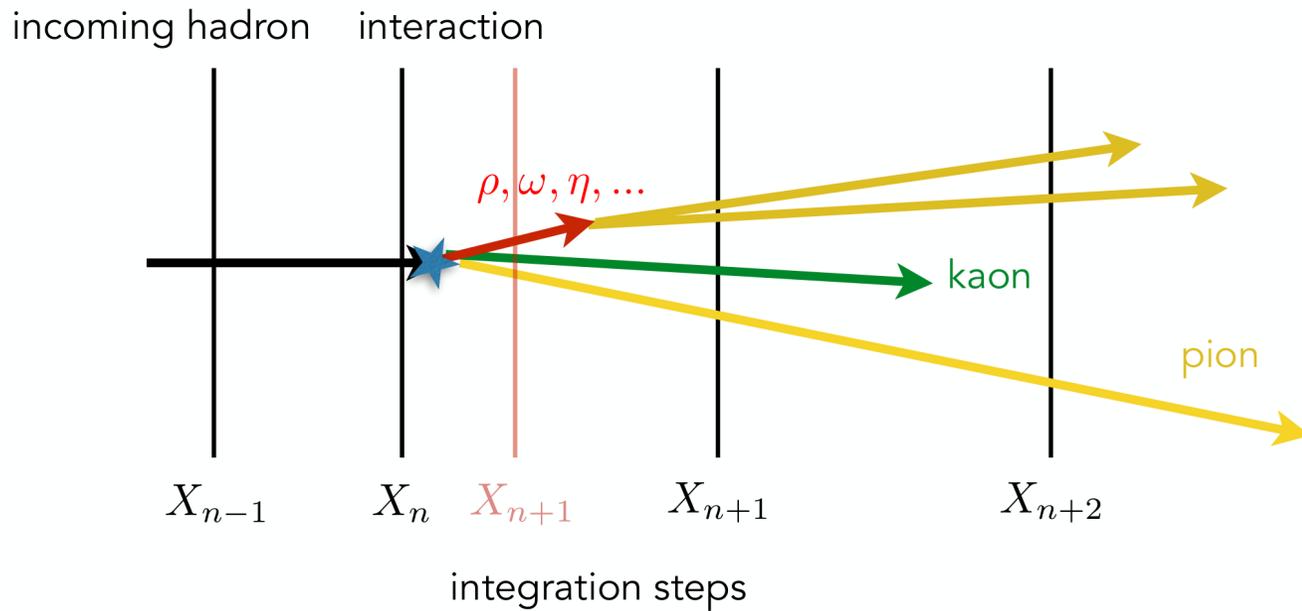
$$p, \bar{p}, n, \bar{n}, \Delta^+, \Delta^{++}, \bar{\Delta}^{++}, \bar{\Delta}^+, \Delta^-, \bar{\Delta}^-, \Delta^0, \bar{\Delta}^0, \Lambda^0, \bar{\Lambda}^0, \Omega^-, \bar{\Omega}^+, \Sigma^{*+}, \bar{\Sigma}^{*-}, \Sigma^{*-}, \bar{\Sigma}^{*+}, \Sigma^{*0}, \bar{\Sigma}^{*0}, \Sigma^+, \bar{\Sigma}^-, \Sigma^0, \bar{\Sigma}^0, \Lambda_C^+, \bar{\Lambda}_C^-, \Omega_C^0, \bar{\Omega}_C^0, \Sigma^-, \bar{\Sigma}^+, \Xi^-, \bar{\Xi}^+, \Xi^0, \bar{\Xi}^0, \Xi_C^+, \bar{\Xi}_C^+, \Xi_C^0, \bar{\Xi}_C^0, \Sigma_C^{*+}, \Sigma_C^{*++}, \bar{\Sigma}_C^{*-}, \bar{\Sigma}_C^{*-}, \Sigma_C^{*0}, \bar{\Sigma}_C^{*0}, \Sigma_C^+, \Sigma_C^{++}, \bar{\Sigma}_C^{--}, \bar{\Sigma}_C^-, \Sigma_C^0, \bar{\Sigma}_C^0, \Xi^{*-}, \bar{\Xi}^{*+}, \Xi^{*0}, \bar{\Xi}^{*0}$$



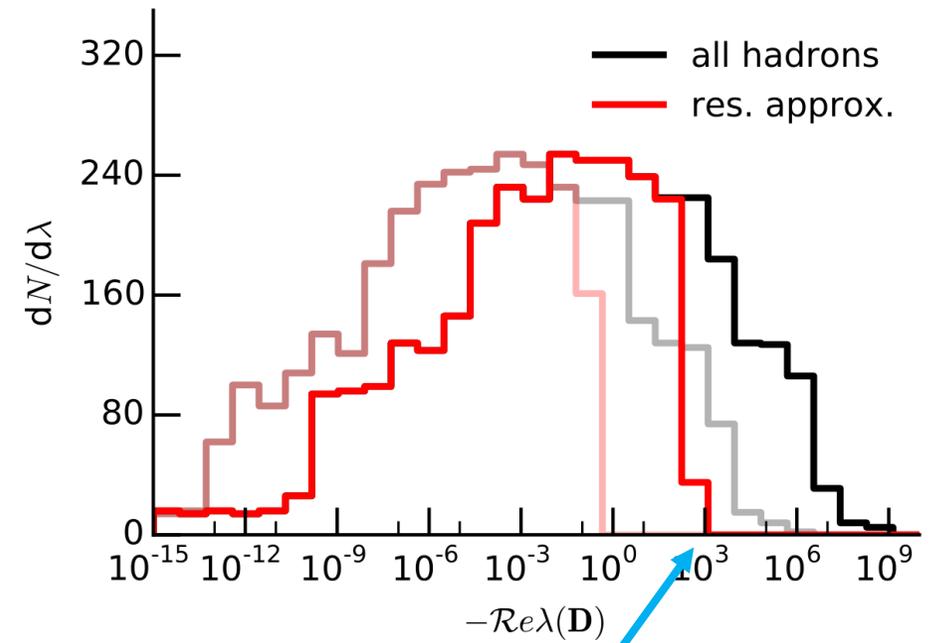
Matrix forms



Stiffness is tricky



Eigenvalues of matrix equation



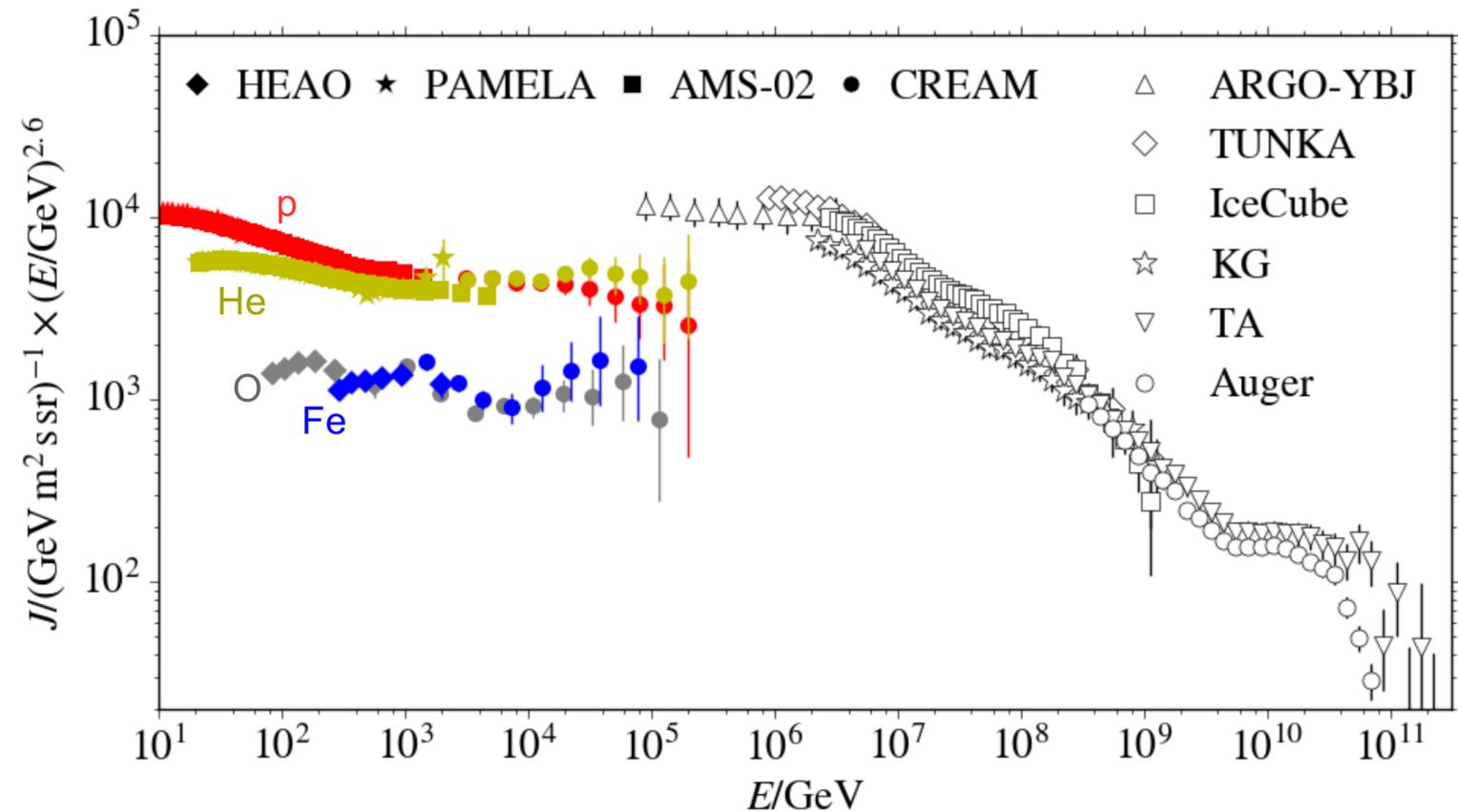
Idea: transport particles only when needed

$$\vec{\Phi}^\omega = \left(\begin{array}{ccc} \lambda_{dec} < t_{mix} \lambda_{int} & & \\ \Phi_{E_0}^\omega & \cdots & \Phi_{E_i}^\omega \\ & \equiv 0 & \\ & \text{treat as} & \\ & \text{resonance} & \end{array} \right) \left| \begin{array}{ccc} \lambda_{dec} \geq t_{mix} \lambda_{int} & & \\ \Phi_{E_{i+1}}^\omega & \cdots & \Phi_{E_N}^\omega \\ & \text{transport as} & \\ & \text{particle} & \end{array} \right)^T$$

Fastest eigenvalue controls integration step



A new take on CR flux uncertainties



- > Combine datasets from direct and indirect observations
- > Use only “golden” datasets with systematic errors (incl. energy scale)
- > Fit whole energy range 10 – 10¹¹ GeV
- > Fit composition:
 - Direct: elements
 - Indirect: mass groups

PoS(ICRC2017)533:

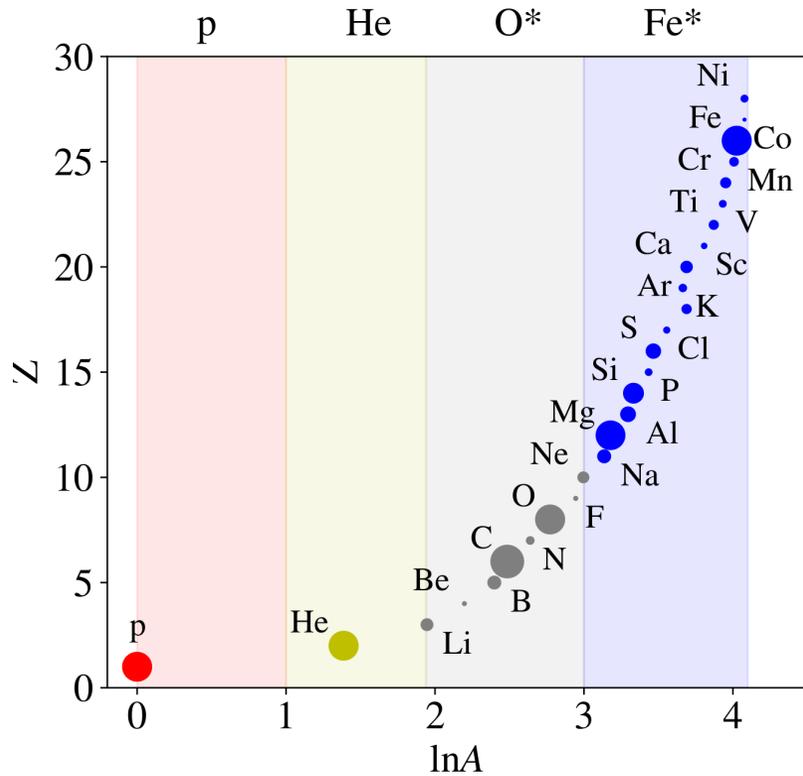
Hans Dembinski, AF, Ralph Engel, Tom K. Gaisser,
Felix Riehn, Todor Stanev

Idea: propagate uncertainties into fluxes
fluxes using partial derivatives from MCEq

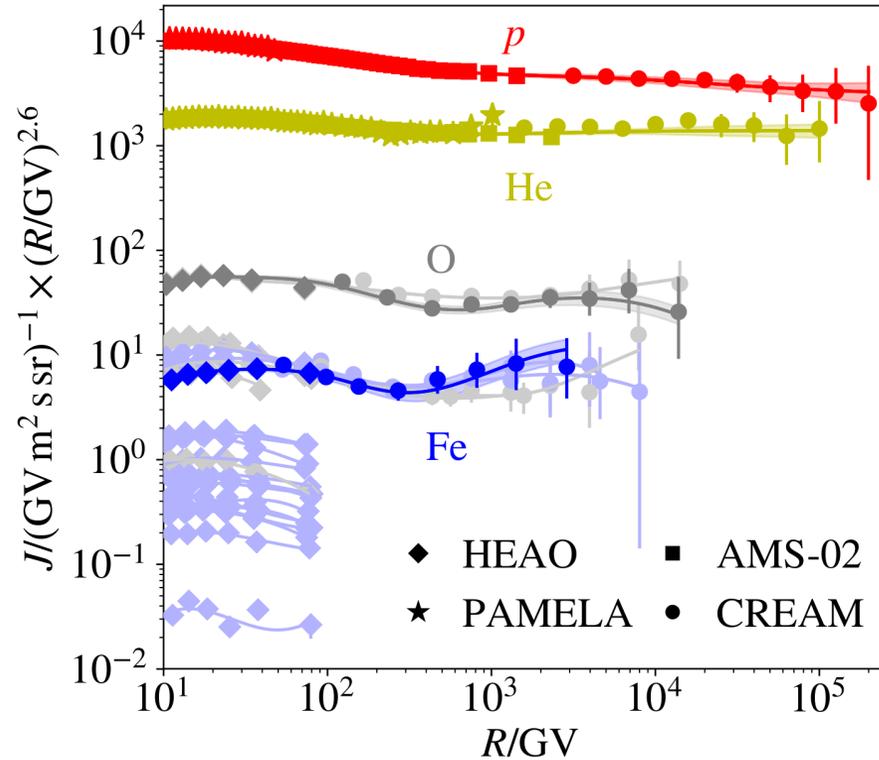


How to create mass groups (indirect) from element fluxes (direct)

Mass group \leftrightarrow Element



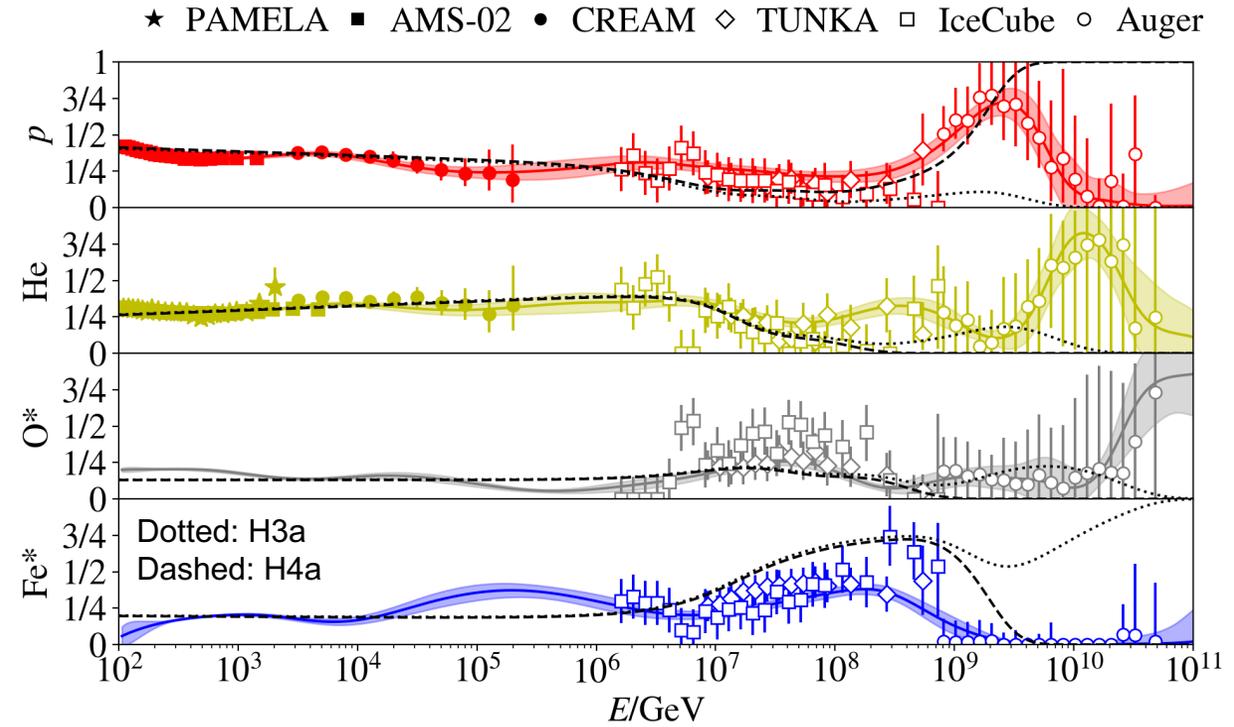
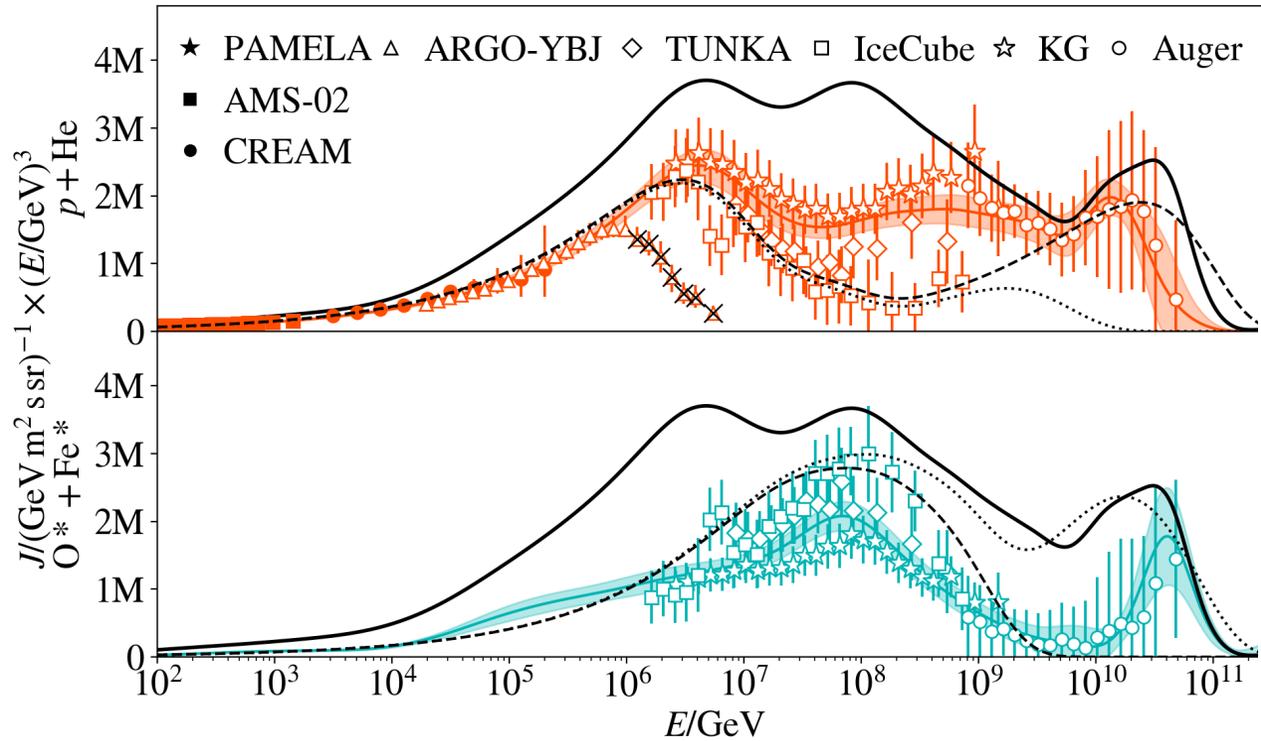
Fit of direct observations



- > Ratio dN/dR roughly equal for neighboring elements of a mass group
 - The only assumption (at all energies)
- > Fit 4 mass groups with distinct sets of B-splines



Reproduction of data from air shower based detectors



GSF: Global Spline Fit

- Data-driven representation of the cosmic ray flux
- Full covariance matrix for all parameters
- Serves as input for flux calculations and error propagation

